AIM 06

Portfolio decision-making

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Portfolio decision-making

- Context:
  - Investment managers have wide array of asset categories (ETFs) from which to choose.
    - Each opportunity can be characterized by its return/risk attributes.
  - How should manager make decision among *n* available asset categories?

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Portfolio decision-making

- Purpose:
  - Review mean-variance portfolio decision-making.
    - Attributable to Markowitz JF (1952).
  - Apply Markowitz mechanics using Excel functions.
    - Solver and AIM add-in functions.
  - Distinguish between minimizing expected risk and maximizing expected return.
    - Measure risk tolerance.

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Portfolio decision-making

- Purpose:
  - Address its practical limitations. Constraints on:
    - Short sales and borrowing securities
    - Borrowing and lending rates
    - Maximum allocations
  - Examine university endowment allocation problem.

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Behavioral assumption

- Decision-making framework arises from single behavioral assumption:
  - Individuals prefer more wealth to less wealth at decreasing rate.
    - Also expressed as *diminishing positive marginal utility of wealth.*
  - Such individuals are said to be *risk averters*.
    - Will never accept *fair bet*.

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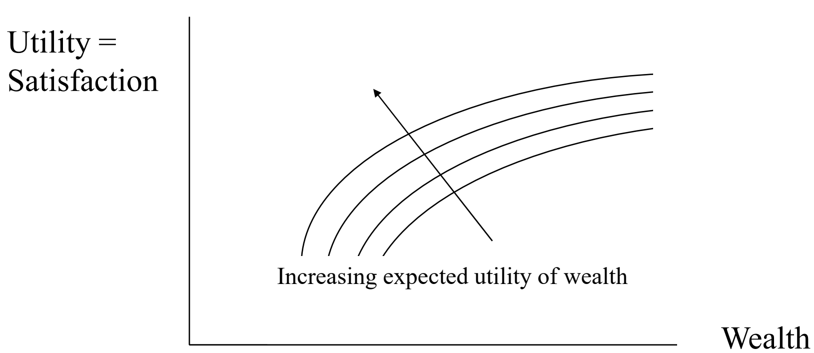
Risk averter's utility function

- Diminishing positive marginal utility of wealth implies utility function is shaped:



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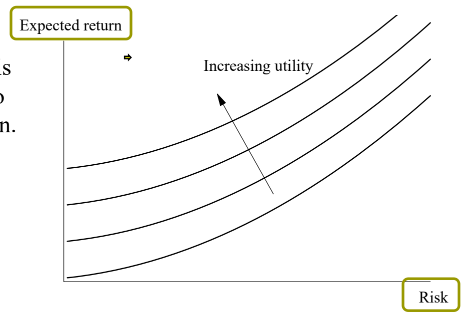
Indifference curves

- Tobin REST (1958) shows that, if individuals have diminishing positive marginal utility of wealth, they have <u>indifference curves</u> shaped like:

Expected return is mean of portfolio return distribution.



Expected risk is standard deviation (or "volatility") of portfolio return distribution.

Also modeled as variance of returns.

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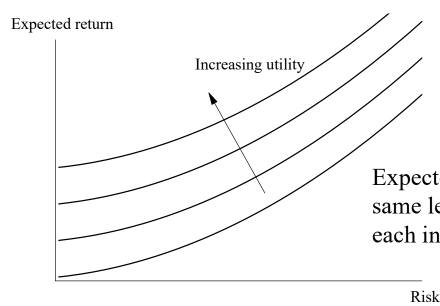
Indifference curves

- If individuals have diminishing positive marginal utility of wealth, they have indifference curves shaped like:



Expected utility is same level along each indifference curve.

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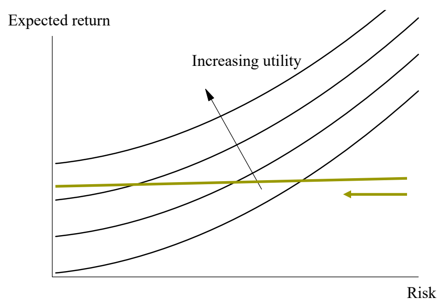
Indifference curves

- If individuals have diminishing positive marginal utility of wealth, they have indifference curves shaped like:

Horizontal line represents portfolios with same level of expected return.

Choose one that minimizes risk to maximize expected utility of wealth.



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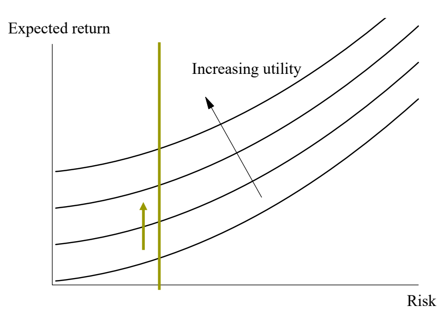
Indifference curves

- If individuals have diminishing positive marginal utility of wealth, they have indifference curves shaped like:

Vertical line represents portfolios with same level of expected risk.

Choose one that maximizes level of expected return to maximize expected utility of wealth.



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Indifference curves

- Based on properties of indifference curves, risk-averters will maximize expected return for given risk level.

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Parameter estimation

- Need to identify available investment opportunities.
- Opportunities are characterized by expected return and risk.
  - Assume n investable asset categories.
  - Must estimate:
    - expected returns – n estimates
    - standard deviations of return – n estimates
    - correlations between each pair of returns – $n(n-1)/2$ estimates

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Parameter estimation

- Notation:

$E_i \equiv$ expected return of asset i

$\sigma_i^2 \equiv Var(R_i) \equiv Cov(R_i,R_i) \equiv$ variance of return of asset i

$\sigma_i \equiv \sqrt{Var(R_i)} \equiv$ standard deviation of return of asset i

$\sigma_{ij} \equiv Cov(R_i, R_j) \equiv \rho_{ij}\sigma_i\sigma_j \equiv$ covariance of returns of assets i and j

$\rho_{ij} \equiv$ correlation of returns of assets i and j

$n \equiv$ number of assets

$X_i \equiv$ proportion of risky asset wealth invested in asset i

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Parameter estimation in practice

- Historical data is generally used to estimate expected volatilities and expected correlations.
- Expected returns are more difficult to estimate.
  - Historical mean return is poor predictor.
  - Rely on consensus estimates of experts.

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Parameter estimation in practice

- ECR Research, Q1 2024, *Expected returns*

Expected Returns

Expected returns are a crucial building block for strategic asset allocation. In-depth research can lead to well-founded expected returns estimates. We believe the analysis of multiple expected returns estimates of experts will, on average, generate better and well documented return estimates, which offer an excellent basis for strategic asset allocation decisions.

In this 'expected returns' report, we bring together the collective intelligence from 47 distinct investment research reports, marking a record in participation, to establish robust consensus forecasts. These consensus forecasts, derived from an unprecedented breadth of data and analysis, are good estimators of expected returns and are available for a large number of asset classes.

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Portfolio expected return and risk

- To compute expected return and expected volatility of portfolio, use AIM add-in functions.
  - Support file: Portfolio decision-making.xlsx
    - Sheet: Minimize risk
  - Code for VBA functions:
    - AIM_PORT_RET
    - AIM_PORT_RISK

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Portfolio expected return and risk

- Given estimates of return and risk for individual securities, portfolio (S) expected return and risk are defined as:
  - Expected return is

$$E_S = \sum_{i=1}^{n} X_i E_i$$

  - Volatility (or standard deviation) of return is

$$\sigma_S = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij} \sigma_i \sigma_j}$$

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Portfolio expected return and risk

- VBA function code: $E_S = \sum_{i=1}^{n} X_i E_i$

```
'**********************************************************************
' Compute portfolio return.
'**********************************************************************
' Variables: x   vector of portfolio weights
'            ret vector of asset returns
'**********************************************************************
' Created: REW,1/3/11
' Revised: REW,1/31/22 Added prefix AIM_ and function description.
'**********************************************************************
Function AIM_Portfolio_return(x, RET)
    n = x.Count
    AIM_Portfolio_return = 0#
    For i = 1 To n
        AIM_Portfolio_return = AIM_Portfolio_return + x(i) * RET(i)
    Next i
End Function
```

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Portfolio expected return and risk

- VBA function code: $\sigma_S = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij} \sigma_i \sigma_j}$

```
'**********************************************************************
' Compute portfolio risk.
'**********************************************************************
' Variables: x   vector of portfolio weights
'            sd  vector of asset return standard deviations
'            rho matrix of asset return correlations
'**********************************************************************
' Created: REW,1/3/11
' Revised: REW,1/31/22 Added prefix AIM_ and function description.
'**********************************************************************
Function AIM_Portfolio_risk(x, sd, rho)
    n = x.Count
    AIM_PORT_RISK = 0#
    For i = 1 To n
        For j = 1 To n
            AIM_Portfolio_risk = AIM_Portfolio_risk + x(i) * x(j) * rho(i, j) * sd(i) * sd(j)
        Next j
    Next i
    AIM_Portfolio_risk = AIM_Portfolio_risk ^ 0.5
End Function
```

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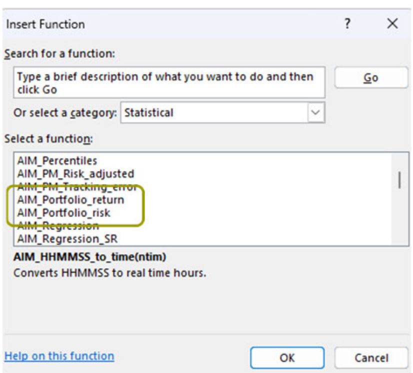
Apply function

- Go to Formulas in main menu.
- Go to Statistical category.



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Minimize risk

- Minimize

$$\text{Volatility} = \sigma_S = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij} \sigma_i \sigma_j}$$

subject to

$$\text{Target expected return constraint: } \sum_{i=1}^{n} X_i E_i = E_S$$

$$\text{Wealth constraint: } \sum_{i=1}^{n} X_i = 1$$

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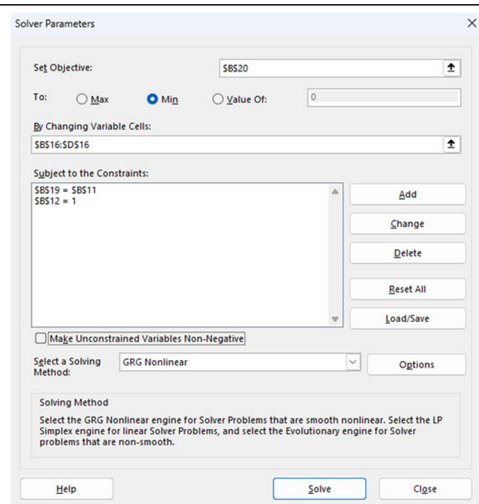
Minimize risk

- Use Solver.



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Minimize risk

- Assumed parameter estimates are:

Risky asset expected return/risk parameters			
	Cash	Bonds	Stocks
Expected return	2%	8%	14%
Volatility	1%	9%	20%
Correlations	Cash	Bonds	Stocks
Cash	1	0.03	-0.01
Bonds	0.03	1	0.05
Stocks	-0.01	0.05	1

- BND is good proxy for US bond market and VTI for stock market.

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Minimize risk

- Optimal allocations are:

Final risky asset portfolio (S)			
	Cash	Bonds	Stocks
Fractional weights	-0.2962	0.9257	0.3705
Dollar investment	-29,617	92,568	37,050
Expected return	12.00%		
Risk	11.42%		

Borrowing (short selling) cash. At what rate?

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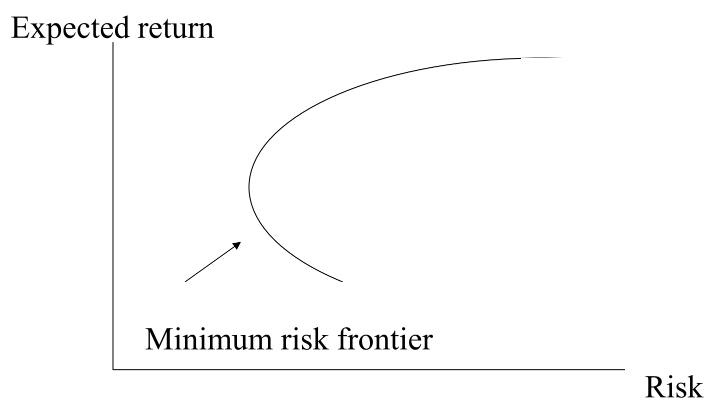
Minimize risk

- Solve problem for different target returns.



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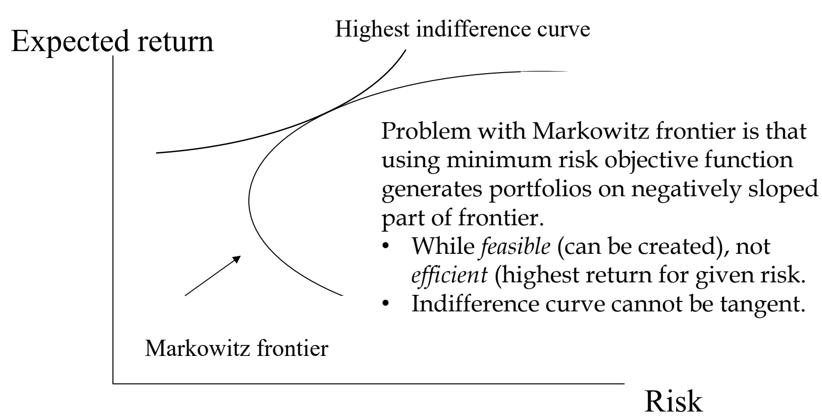
Minimize risk

- Solve problem for different target returns.



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Short sales constraint

- Reality is that there are constraints to short sales.
  - For small investors, short sales may be restricted by broker.
  - For many large investors (institutions) like endowments or pension funds, short sales are barred.
  - Even for large investors that are not barred, short selling is costly.
- Assume short sales are prohibited (i.e., *X*'s cannot be negative).

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Minimize risk (no short sales)

- Minimize

$$\text{Volatility} = \sigma_S = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij} \sigma_i \sigma_j}$$

subject to:

$$\text{Target expected return constraint: } \sum_{i=1}^{n} X_i E_i = E_S$$

$$\text{Wealth constraint: } \sum_{i=1}^{n} X_i = 1$$

$$\text{Non-negativity constraints: } X_i \geq 0,\ i = 1, \ldots, n$$

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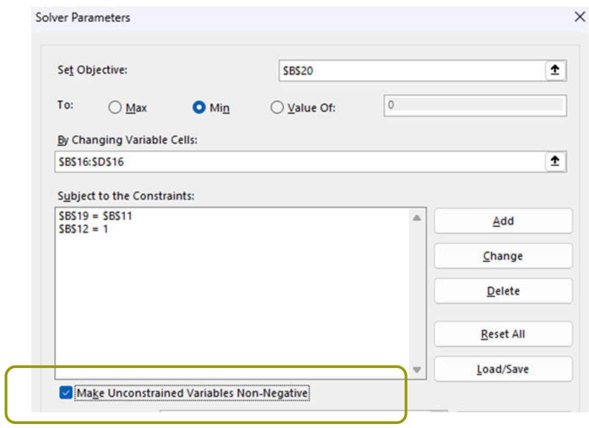
Minimize risk (no short sales)

- Impose non-negativity constraint.



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Maximize return

- Problem can be re-specified to maximize expected return for given level of risk.
  - Individual specifies risk tolerance (i.e., highest level of volatility he/she is willing to tolerate).

$$\text{Risk tolerance} \equiv \sigma_{RT}$$

  - Support file: Understanding risk tolerance.xlsx

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Risk tolerance

- Setting level of risk tolerance is unintuitive.
- Deducing level of risk tolerance is possible by eliciting loss aversion statements such as:
  "I am willing to tolerate a loss of greater than TL%
  if the chance of the loss is less than PL%."

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Risk tolerance

- <u>Support file</u>: Risk tolerance.xlsx
  - Assume ln returns are normally distributed.

	A	B	C
1	**Risk tolerance given loss aversion**		
2	Tolerable loss	*TL*	-20.0%
3	Chance of loss	*PL*	2.5%
4	Mean (annualized)	μ	5.00%
5	Risk tolerance	*RT*	12.76%

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Factors affecting risk tolerance

- Utility function/indifference curves
  - Slope and curvature measure degree of risk aversion.
- Investment horizon. Examples include:
  - Planning for retirement in 40 years.
  - Infrastructure spending from collection of state taxes.
  - University endowment spending.

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Maximize return

- Problem can be re-specified to maximize expected return for given level of risk.
  - Individual specifies risk tolerance (i.e., highest level of volatility he/she is willing to tolerate).

$$\text{Risk tolerance} \equiv \sigma_{RT}$$

  - Support file: Portfolio decision-making.xlsx
    - Sheet: Maximize return

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Maximize return

- Maximize

$$E_S = \sum_{i=1}^{n} X_i E_i$$

subject to:

Risk tolerance constraint: $\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij}\sigma_i\sigma_j} \le \sigma_{RT}$

Wealth constraint: $\sum_{i=1}^{n} X_i = 1$

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Maximize return (no short sales)

- Maximize

$$E_S = \sum_{i=1}^{n} X_i E_i$$

subject to:

Risk tolerance constraint: $\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij}\sigma_i\sigma_j} \le \sigma_{RT}$

Wealth constraint: $\sum_{i=1}^{n} X_i = 1$

Non-negativity constraints: $X_i \ge 0,\ i = 1,\ldots,n$

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Other constraints

- Reality is borrowing rate is higher than lending rate.
  - Quoted as number of basis points above lending rate (e.g., 100 bps above Fed funds rate).
    - Replace expected return on cash with higher rate.
- Reality is amount of borrowing is constrained.
  - Under Federal Reserve Regulation T, maximum leverage ratio is 50%
    - E.g., if investor has $10,000 available to buy securities, she can borrow up to $10,000 on margin.
    - Constrain allocation to cash to be at least -1.00.

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Endowment allocation decision

- Endowments are:
  - Assets (e.g., cash, stocks, bonds) set aside for long-term benefit of organization.
    - Original amount known as "corpus."
  - Permanent and will generate income forever.
    - Amount of corpus is preserved.

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Endowment allocation decision

- Problem: Endowment management
  - Universities manage endowments in manner to maximize return subject to constraint principal does not fall (i.e., endowment lives in perpetuity).
  - Income contributes to operating budget.
    - Pays fixed % to operating budget.
    - Excess return reinvested in endowment (e.g., endowment principal grows).

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Endowment allocation decision

- Problem: Endowment management
  - Vanderbilt's endowment is about $6.5 billion.
  - In making decision regarding payout, risk tolerance is set to 10.3%.
  - About 4.25% of principal is provided to annual operating budgets of schools within university.
  - Examine decision made in late 2000s.

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Endowment allocation decision

- Support file: Endowment allocation decision.xlsx

Endowment asset allocation			
Asset class	Expected return	Expected risk	Maximum allocation
Global equities (GE)	8.90%	17.40%	50.00%
Private equity (PE)	12.60%	24.00%	12.00%
Venture capital (VC)	14.00%	28.00%	6.00%
Non-marketable energy (NME)	8.20%	22.00%	4.00%
Non-marketable real estate (NMRE)	7.80%	13.40%	10.00%
Natural resources: Other (NRO)	7.80%	12.40%	2.00%
Absolute return (AR)	6.00%	6.20%	20.00%
Long Treasury (LT)	4.20%	8.30%	10.00%
			114.00%

Correlations	GE	PE	VC	NME	NMRE	NRO	AR	LT
Global equities (GE)	1.0	0.5	0.3	0.1	0.3	0.0	0.4	0.1
Private equity (PE)	0.5	1.0	0.5	-0.1	0.1	0.3	0.3	-0.1
Venture capital (VC)	0.3	0.5	1.0	-0.1	0.0	0.1	0.2	-0.1
Non-marketable energy (NME)	0.1	-0.1	-0.1	1.0	0.1	0.2	0.1	-0.1
Non-marketable real estate (NMRE)	0.3	0.1	0.0	0.1	1.0	0.1	0.4	0.1
Natural resources: Other (NRO)	0.0	0.3	0.1	0.2	0.1	1.0	0.2	0.2
Absolute return (AR)	0.4	0.3	0.2	0.1	0.4	0.2	1.0	-0.1
Long Treasury (LT)	0.1	-0.1	-0.1	-0.1	0.1	0.2	-0.1	1.0

Investment office sets expected returns, risks and correlations.

Investment committee sets maximum allocations.

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Lesson summary

- Portfolio decision-making is based on Markowitz (1952) return-risk mechanics for risky asset categories.
  - Easily solved using Solver.
- Objective function: Maximize return given risk tolerance.

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Lesson summary

- Generalize portfolio decision-making to include constraints on:
  - Short sales and borrowing.
  - Borrowing and lending rates.
  - Maximum allocations.
- Examine actual endowment allocation problem.

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