CAPM REVIEW

The Sharpe (1964)/Lintner (1965) capital asset pricing model (CAPM)[1] is based on the Markowitz (1952)/Tobin (1958) portfolio theory, in which all individuals evaluate risky investments solely in terms of the mean and variance of the future return distribution. The CAPM extends the framework by considering all individuals' demands collectively. It assumes that all individuals share the same beliefs about the levels of the expected returns, standard deviations (variances), and correlations (covariances) of security returns and that all individuals can borrow and lend at the same risk-free rate of interest. The model has three main results: (a) the capital market line (CML), (b) the composition of the market portfolio, and (c) the security market line (SML).

CAPM - Capital market line

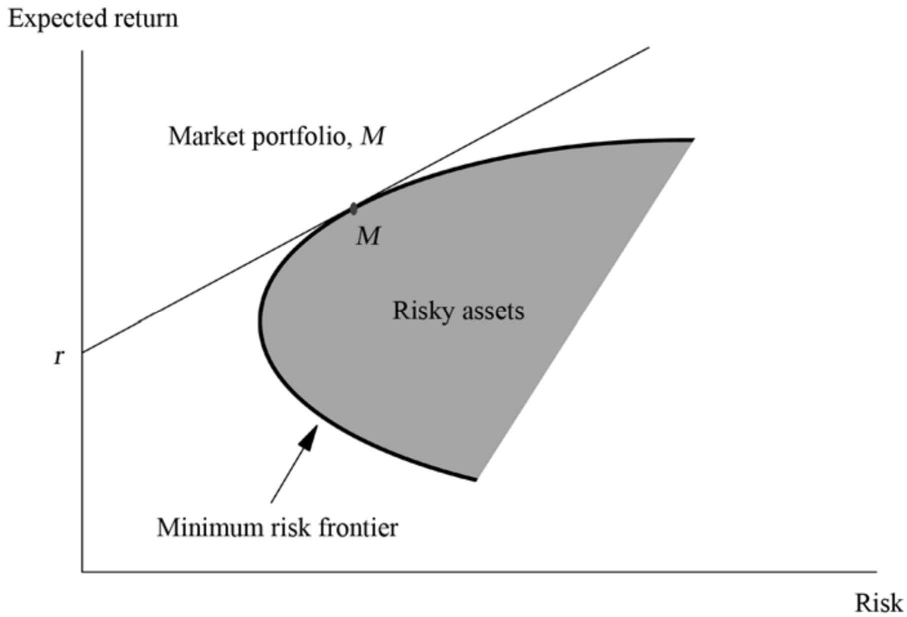
The *capital market line* (CML) represents the relation between expected return and risk for efficient portfolios. With common expectations regarding the expected returns, standard deviations, and correlations for risky assets and risk-free borrowing and lending, all individuals hold the same risky asset tangency portfolio *M* combined with the risk-free asset. See Figure 1. Since *M* must contain all risky assets in the economy, it is called the *market portfolio*. More formally, the CML is

$$E_P = r + \left(\frac{E_M - r}{\sigma_M} \right) \sigma_P , \tag{1}$$

where E_P and E_M are the expected returns on the individual's and market portfolios, σ_P and σ_M are their volatilities, and r is the risk-free return. An individual's allocation between the market portfolio and the risk-free asset depends on his degree of risk aversion. A risk-minimizer will invest all his wealth in a risk-free asset. A risk-averse individual will choose a portfolio along the *capital market line* (1). If his highest indifference curve is tangent to the left of *M*, the *optimal* portfolio will be a *lending* portfolio—some wealth invested in *M* and some in the risk-free asset. Suppose it is tangent to the right of *M*. In that case, the optimal portfolio will be a *borrowing* portfolio—not only is all wealth invested in *M*, but also additional funds are borrowed and invested in *M*.

[1] Jack L. Treynor independently developed the CAPM in a working paper dated 1962. Unfortunately, his paper was never published.

Figure 1: Minimum variance frontier with risk-free borrowing and lending and common return expectations.



CAPM - Composition of the market portfolio

Within the CAPM framework, creating portfolio *M* can be done more intuitively. First, we know that if the market is in equilibrium, the total demand by individuals for risky asset *i* must equal the total supply of asset *i*, that is,

$$\sum_{k=1}^{m} X_i^k w^k = V_i \,, \tag{2}$$

where *k* represents the *k*th individual, *m* represents the number of individuals in the market, X_i^k is the proportion of *k*'s risky asset wealth, w^k, invested in asset *i*, and V_i is the market value of asset *i*. Second, we know that, in equilibrium, total demand by individuals for risky assets must equal total supply, that is,

$$\sum_{k=1}^{m} w^k = \sum_{i=1}^{n} \sum_{k=1}^{m} X_i^k w^k = \sum_{i=1}^{n} V_i = V_M \,, \tag{3}$$

where *n* is the number of risky assets in the market and V_M is the market value of all risky assets. Finally, we know (from the work of Markowitz (1952) that the allocation among risky securities in creating *M* (or any portfolio along the efficiency frontier) is unique. All investors allocate their risky asset wealth in the same proportions, that is, $X_i^k = X_i$ for all *k*. From (2) and (3), the optimal proportion of risky asset of wealth invested in risky asset *i* is

$$X_i = \frac{V_i}{V_M}, \tag{4}$$

the market value of risky asset *i* divided by the market value of all risky assets.

Note that (4) is established without regard to the risky assets. Theoretically, the set of assets should be all-inclusive (e.g., stocks, bonds, and commodities). In practice, the S&P 500 stock index usually represents the market. While there may have been practical reasons why this might have been appropriate in the past,[2] it is not appropriate today. The S&P 500 portfolio does not include all US stocks.[3] A total US stock market index like the CRSP Total Market Value Index, CRSPTMT, or the S&P Total Market Index, S&P TMI, is more appropriate. Indeed, over the years, the S&P 500 has become dominated by technology stocks like Apple and Microsoft, which account for more than 10% of the index value. In that sense, it has become a "thematic index."[4] But, more importantly, the benchmark should be all-inclusive and include other risky assets like bonds, commodities, and the like.

CAPM - Security market line

The *security market line* (SML) represents risky assets' equilibrium expected return/risk relation. To identify this relation, we recognize asset *i*'s contribution to the market portfolio's expected excess return and risk. The market portfolio expected excess return and risk are

$$E_M - r = \sum_{i=1}^{n} X_i \left(E_i - r \right) \tag{5}$$

and

$$\sigma_M^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij} \ . \tag{6}$$

The marginal contribution of asset *i* to the expected excess return of the market portfolio is

$$\frac{\partial E_M - r}{\partial X_i} = E_i - r \ , \tag{7}$$

and the marginal contribution of asset *i* to the risk of the market portfolio is

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[2] The CAPM assumes that individuals can buy or sell risky assets freely and, in the case of short sales, have full use of proceeds. With that being the case, the market portfolio can be bought or sold freely. Historically, commissions and bid/ask spreads on stocks were high, and short sales by individuals were encumbered. The S&P 500 represented a well-known market-cap index comprised of highly liquid stocks.

[3] Indeed, the S&P 500 index is not truly passive. Its membership is decided by an index committee at Standard and Poor's and changes periodically through time.

[4] Thematic indexes are subsets of stocks focused on a particular theme (e.g., technology, ESG, high dividend yield).

$$\frac{\partial \sigma_M^2}{\partial X_i} = \sum_{j=1}^{n} X_j \sigma_{ij} = \sigma_{iM} \ . \tag{8}$$

In equilibrium, all risky assets must have the same expected excess return/risk tradeoff; hence,

$$\frac{E_i - r}{\sigma_{iM}} = \frac{E_M - r}{\sigma_M^2} \ . \tag{9}$$

The term on the right-hand side of (9) is the *market price of risk*. Rearranging,

$$E_i = r + \left(E_M - r\right)\frac{\sigma_{iM}}{\sigma_M^2} = r + \left(E_M - r\right)\beta_i \, . \tag{10}$$

Equation (10) is called the *security market line* (SML). The SML represents the equilibrium expected return/risk relation for all risky securities in the marketplace. And, if (10) holds for all risky securities, it holds for portfolios.

References and suggested readings

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