


Managing risk dynamically

Market maker

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Managing risk dynamically

- Context:
 - Market liquidity providers (e.g., exchange market makers, OTC option dealers) earn revenue from bid/ask spread.
 - Must manage risk of inventory.
- Purpose:
 - Show how option price risk is managed through time.
 - Use risk measures developed in last section.

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Dynamic risk management

- Dynamic risk management has three steps.
 - Identify portfolio's risk exposures.
 - Identify available hedge instruments.
 - Identify lowest cost hedge.

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Stock option market maker

- Problem information:
 - Market maker for ABC option class.
 - Assume:
 - ABC's stock pays no dividends, has a current price of \$75 and has a volatility rate is 40%.
 - Interest rate is 5%.
 - Stock options are European-style.

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Market maker

- Problem information:
 - Assume following option series are listed.

Option series in class							
Option series			Value	Delta	Gamma	Vega	Rho - r
Exercise price	(C)all/ (P)ut	Years to expiration					
65	c	0.250	12.536	0.810	0.018	10.175	12.054
70	c	0.250	9.143	0.694	0.023	13.153	10.728
75	c	0.250	6.414	0.565	0.026	14.764	8.982
80	c	0.250	4.336	0.436	0.026	14.770	7.098
85	c	0.250	2.832	0.322	0.024	13.438	5.321
65	p	0.250	1.729	-0.190	0.018	10.175	-3.994
70	p	0.250	3.274	-0.306	0.023	13.153	-6.554
75	p	0.250	5.483	-0.435	0.026	14.764	-9.535
80	p	0.250	8.343	-0.564	0.026	14.770	-12.654
85	p	0.250	11.777	-0.678	0.024	13.438	-15.665

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Market maker

- Problem information:
 - Supporting file: ABC market maker.xlsx

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Market maker

- Problem information:
 - Market maker's current position:
 - short 30 65-calls
 - short 50 65-puts
 - short 10 80-calls
 - What are her net risk exposures?

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Market maker

- Compute risks of current position.
 - Multiply each option's Greek by number of contracts.
 - Sum across options.

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Market maker

□ Problem information:

- Net risk exposures are:

Current position of market maker									
Exercise price	Option series		No. of contracts (+ long/-short)	Value	Delta	Gamma	Vega	Rho	
	(C)all/(P)ut	Years to expiration							
65	c	0.25	-30	-376.085	-24.301	-0.543	-305.257	-361.620	
65	p	0.25	-50	-86.436	9.499	-0.904	-508.762	199.707	
80	c	0.25	-10	-43.363	-4.364	-0.263	-147.696	-70.978	
Unhedged portfolio risk attributes				-505.885	-19.166	-1.710	-961.715	-232.891	

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Market maker

□ Problem information:

- Net risk exposures are:

Current position of market maker									
Exercise price	Option series		No. of contracts (+ long/-short)	Value	Delta	Gamma	Vega	Rho	
	(C)all/(P)ut	Years to expiration							
65	c	0.25	-30	-376.085	-24.301	-0.543	-305.257	-361.620	
65	p	0.25	-50	-86.436	9.499	-0.904	-508.762	199.707	
80	c	0.25	-10	-43.363	-4.364	-0.263	-147.696	-70.978	
Unhedged portfolio risk attributes				-505.885	-19.166	-1.710	-961.715	-232.891	

Market maker is short options. Earns interest on cash.

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Market maker

- Problem information:
 - Net risk exposures are:

Current position of market maker								
Exercise price	Option series		No. of contracts	Value	Delta	Gamma	Vega	Rho
	(C)all/ (P)ut	Years to expiration	(+ long/ -short)					
65	c	0.25	-30	-376.085	-24.301	-0.543	-305.257	-361.620
65	p	0.25	-50	-86.436	9.499	-0.904	-508.762	199.707
80	c	0.25	-10	-43.363	-4.364	-0.263	-147.696	-70.978
Unhedged portfolio risk attributes				-505.885	-19.166	-1.710	-961.715	-232.891

Market maker is implicitly short 19.166 shares of stock.

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Market maker

- Problem information:
 - Suppose market maker wants to hedge her delta exposure using 75-calls. How many calls should she buy?

$$19.166 / 0.565 = 33.92$$

Now, what are her exposures?

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Market maker

- Problem information:
 - New net risk exposures:

Market maker portfolio									
Option series			No. of contracts	Value	Delta	Gamma	Vega	Rho	
Exercise price	(C)all/ (P)ut	Years to expiration	(+ long/ -short)						
65	c	0.25	-30	-376.085	-24.301	-0.543	-305.257	-361.620	
65	p	0.25	-50	-86.436	9.499	-0.904	-508.762	199.707	
80	c	0.25	-10	-43.363	-4.364	-0.263	-147.696	-70.978	
Unhedged portfolio risk attributes				-505.885	-19.166	-1.710	-961.715	-232.891	
Hedge instruments									
75	c	0.25	33.95	217.768	19.166	0.891	501.234	304.920	
Hedged portfolio risk attributes				-288.117	0.000	-0.819	-460.480	72.029	

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Market maker

- Problem information:
 - Suppose market maker wants to hedge her delta and vega exposures.
 - To hedge two different exposures, two options are needed.
 - Assume she uses 75-call and 75-put.

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Market maker

□ Problem information:

- Solve analytically.

$$\text{Delta hedge: } 19.166 = .565n_c + (-.435)n_p$$

$$\text{Vega hedge: } 961.715 = 14.764n_c + 14.764n_p$$

$$n_c = 47.53 \text{ and } n_p = 17.61$$

- Solve numerically (e.g., SOLVER).

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Market maker

□ Problem information:

- New net risk exposures:

Market maker portfolio									
Exercise price	Option series		No. of contracts (+ long/-short)	Value	Delta	Gamma	Vega	Rho	
	(C)all/(P)ut	Years to expiration							
65	c	0.25	-30	376.085	-24.301	-0.543	-305.257	-361.620	
65	p	0.25	-50	86.436	9.499	-0.904	-508.762	199.707	
80	c	0.25	-10	43.363	-4.364	-0.263	-147.696	-70.978	
Unhedged portfolio risk attributes				505.885	-19.166	-1.710	-961.715	-232.891	
Hedge instruments									
75	c	0.25	47.53	-304.886	26.833	1.248	701.753	426.904	
75	p	0.25	17.61	-96.539	-7.667	0.462	259.961	-167.898	
Hedged portfolio risk attributes				104.460	0.000	0.000	0.000	26.115	

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Index option market maker

- Illustration: Suppose market maker in S&P 500 index options ends the day with:

Option series			No. of contracts
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)
900	c	30	-100
950	c	30	-200
1000	c	90	-150
900	p	90	50
950	p	360	-100
1000	p	720	-200
1100	c	720	-100

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Index option market maker

- Illustration: Assume he wants to hedge delta risk overnight.
- Consider two hedging alternatives.
 - use S&P 500 futures or 975-call options
 - S&P 500 index:
 - current level 1,000
 - dividend yield 2%
 - volatility rate 20%
 - S&P 500 futures:
 - 90 days to expiration
 - current price at full carry, 1004.94
 - delta 1.0049.
 - S&P 975-call (European-style):
 - 90 days to expiration
 - current price 55.432
 - delta 0.635
 - Risk-free interest rate 4%
 - Identify the number of contracts to enter in each case.

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Index option market maker

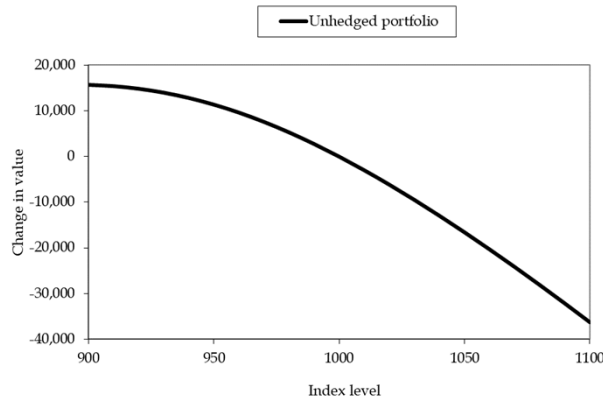
- Supporting file: S&P 500 market maker.xls

Market maker portfolio									
Exercise price	Option series		No. of contracts (+ long/-short)	Value	Delta	Gamma	Vega	Rho	
	(C)all/(P)ut	Days to expiration							
900	c	30	-100	10,196.59	-96.94	-0.1154	-1,896.44	-7,129.19	
950	c	30	-200	11,367.74	-165.61	-0.8831	-14,516.94	-12,677.73	
1000	c	90	-150	6,271.21	-80.54	-0.5966	-29,423.39	-18,311.65	
900	p	90	50	-313.98	-6.12	0.1020	5,028.22	-1,586.17	
950	p	360	-100	4,601.73	31.76	-0.1774	-34,996.41	35,859.39	
1000	p	720	-200	17,546.70	74.87	-0.2625	-103,560.05	182,293.85	
1100	c	720	-100	8,500.74	-45.83	-0.1363	-53,771.89	-73,630.18	

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Index option market maker

- Delta-exposure



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Index option market maker

- Use futures.

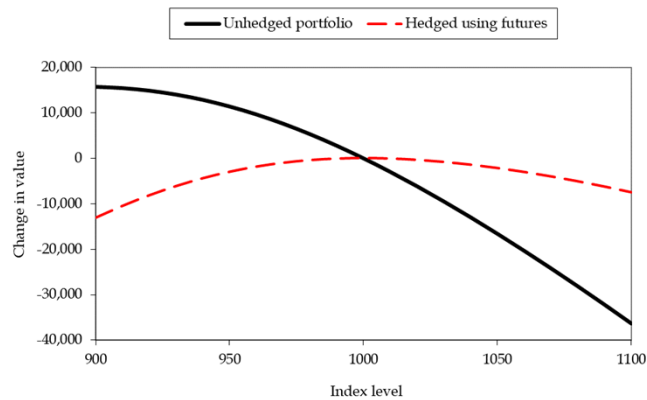
$$n_F = \frac{288.41}{1.0049} = 286.99$$

Option series			No. of contracts	Value	Delta	Gamma	Vega	Rho
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)					
Unhedged portfolio				58,170.73	-288.41	-2.07	-233,136.90	104,818.32
Hedge instruments								
	F	90	286.99	0.00	288.41	0.00	0.00	71,114.22
Hedged portfolio				58,170.73	0.00	-2.07	-233,136.90	175,932.54

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Index option market maker

- Use futures



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Index option market maker

- Use 975-call.

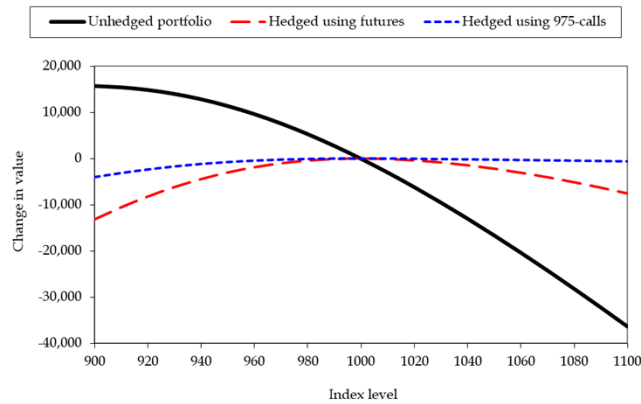
$$n_{975 \text{ call}} = \frac{288.41}{0.635} = 453.98$$

Option series			No. of contracts	Value	Delta	Gamma	Vega	Rho
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)					
Hedge instruments				58,170.73	-288.41	-2.07	-233,136.90	104,818.32
975	c	90	453.98	-25,165.31	288.41	1.7043	84,049.43	64,909.08
Hedged portfolio				33,005.43	0.00	-0.37	-149,087.47	169,727.40

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Index option market maker

- Use 975-call



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Index option market maker

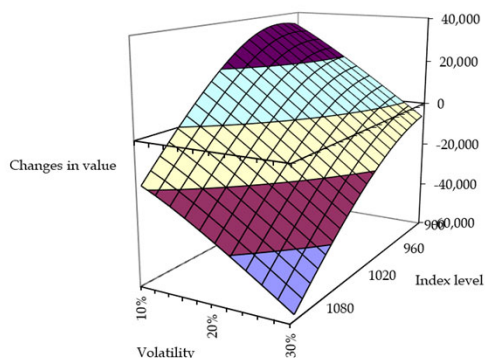
- Illustration: Assume he wants to hedge delta and vega risks overnight.
 - Use S&P 500 futures and 975-call options

Option series			No. of contracts (+ long/ -short)	Value	Delta	Gamma	Vega	Rho
Exercise price	(C)all/ (P)ut	Days to expiration						
Unhedged portfolio				58,170.73	-288.41	-2.07	-233,136.90	104,818.32

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Index option market maker

- Illustration: Delta-vega exposures unhedged.



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Index option market maker

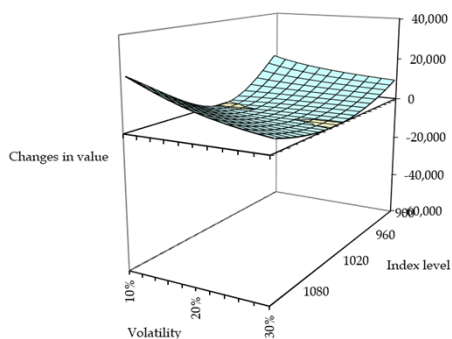
- Illustration: Assume he wants to hedge delta and vega risks overnight. Solve analytically.

Option series			No. of contracts (+ long/ -short)	Value	Delta	Gamma	Vega	Rho
Exercise price	(C)all/ (P)ut	Days to expiration						
Unhedged portfolio				58,170.73	-288.41	-2.07	-233,136.90	104,818.32
Hedge instruments								
	F	90	-509.06	0.00	-511.58	0.00	0.00	-126,142.89
975	c	90	1,259.27	-69,803.70	799.99	4.7275	233,136.90	180,045.24
Hedged portfolio				-11,632.97	0.00	2.66	0.00	158,720.67

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Index option market maker

- Illustration: Assume he wants to hedge delta and vega risks overnight. After hedge is in place:



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Index option market maker

- Illustration: Assume he wants to hedge delta and vega risks overnight. The costs/benefits of hedge instrument positions include:
 - Bid/ask spreads and commissions:
 - Assume commission rate of \$5 per contract.
 - Opportunity cost of funds
 - Assume lending/borrowing rate of 4%.
 - Erosion in time premium of options
 - Identify least cost hedge portfolio assuming his hedging horizon is one day.

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Index option market maker

- Illustration: Assume he wants to hedge delta and vega risks overnight.
 - Available hedge instruments.

Potential hedge instruments			
(F)utures/			
Exercise price	(C)all/ (P)ut	Days to expiration	Price
	F	90	1,004.94
975	c	90	25.78
975	p	90	55.43
1025	c	90	50.52
1025	p	90	30.66

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Index option market maker

□ Illustration: Model costs over hedge period.

Option series			No. of contracts (+ long/ -short)	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Exercise price	(C)all/ (P)ut	Days to expiration								
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hedged portfolio				58,170.73	-288.41	-233,136.90	122.82	0.00	0.00	0.00
Total costs									0.00	

$$\text{Trading costs} = |\# \text{ of contracts}| \times \text{commission rate}$$

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Index option market maker

□ Illustration: Model costs over hedge period.

Option series			No. of contracts (+ long/ -short)	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Exercise price	(C)all/ (P)ut	Days to expiration								
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hedged portfolio				58,170.73	-288.41	-233,136.90	122.82	0.00	0.00	0.00
Total costs									0.00	

$$\text{Interest cost} = \text{Value} \times (e^{r\Delta t} - 1)$$

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Index option market maker

□ Illustration: Model costs over hedge period.

Option series			No. of contracts	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)							
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hedged portfolio				58,170.73	-288.41	-233,136.90	122.82	0.00	0.00	0.00

Total costs 0.00

$$\text{Time erosion} = \text{Theta} \times \Delta t$$

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Index option market maker

□ Illustration: Use SOLVER to identify hedge.

Option series			No. of contracts	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)							
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	-1.56	0.00	-1.57	0.00	-31.33	-7.80		-0.09
975	c	90	65.14	-3,610.74	41.38	12,059.50	-5,573.99	-325.69	-0.40	-15.27
975	p	90	0.00	-0.04	0.00	0.30	-0.11	-0.01	0.00	0.00
1025	c	90	884.17	-27,106.02	387.69	172,360.09	-76,571.19	-4,420.85	-2.97	-209.78
1025	p	90	249.91	-12,624.46	-139.10	48,717.02	-16,470.51	-1,249.54	-1.38	-45.12
Hedged portfolio				14,829.47	0.00	0.00	-98,524.32	-6,003.88	-4.75	-270.27

Total costs 6,278.89

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Index option market maker

□ Illustration: Use SOLVER to identify hedge.

Option series			No. of contracts	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)							
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	-1.56	0.00	-1.57	0.00	-31.33	-7.80		-0.09
975	c	90	65.14	-3,610.74	41.38	12,059.50	-5,573.99	-325.69	-0.40	-15.27
975	p	90	0.00	-0.04	0.00	0.30	-0.11	-0.01	0.00	0.00
1025	c	90	884.17	-27,106.02	387.69	172,360.09	-76,571.19	-4,420.85	-2.97	-209.78
1025	p	90	249.91	-12,624.46	-139.10	48,717.02	-16,470.51	-1,249.54	-1.38	-45.12
Hedged portfolio				14,829.47	0.00	0.00	-98,524.32	-6,003.88	-4.75	-270.27

Total costs 6,278.89

Could drop these contracts from feasible set or simply run Solver again.

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Index option market maker

□ Illustration: Run again.

Option series			No. of contracts	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)							
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	0.00	0.00	0.00	0.00	0.03	-0.01		0.00
975	c	90	63.84	-3,539.00	40.56	11,819.90	-5,463.24	-319.22	-0.39	-14.97
975	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	c	90	884.11	-27,104.12	387.67	172,348.02	-76,565.83	-4,420.54	-2.97	-209.77
1025	p	90	251.20	-12,689.75	-139.82	48,968.98	-16,555.70	-1,256.00	-1.39	-45.36
Hedged portfolio				14,837.86	0.00	0.00	-98,461.93	-5,995.76	-4.75	-270.10

Total costs 6,270.61

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Index option market maker

□ Illustration: Ran five times in all.

Option series			No. of contracts							
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
	F	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	c	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
975	p	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1025	c	90	958.78	-29,393.46	420.41	186,905.38	-83,032.96	-4,793.92	-3.22	-227.49
1025	p	90	237.16	-11,980.37	-132.00	46,231.53	-15,630.21	-1,185.79	-1.31	-42.82
Hedged portfolio				16,796.90	0.00	0.00	-98,540.34	-5,979.70	-4.53	-270.31
Total costs									6,254.55	

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Index option market maker

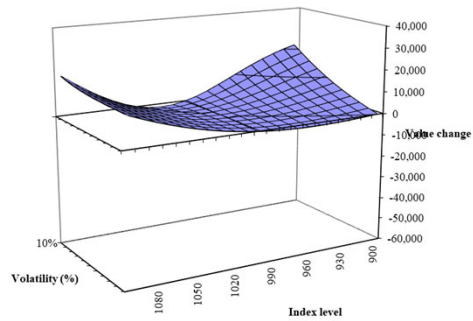
□ Illustration: Drop contracts and re-solve.

Option series			No. of contracts							
Exercise price	(C)all/ (P)ut	Days to expiration	(+ long/ -short)	Value	Delta	Vega	Theta	Trading costs	Interest cost	Time erosion
Unhedged portfolio				58,170.73	-288.41	-233,136.90	122.82			
Hedge instruments										
1025	c	90	958.78	-29,393.47	420.41	186,905.40	-83,032.97	-4,793.92	-3.22	-227.49
1025	p	90	237.16	-11,980.36	-132.00	46,231.50	-15,630.20	-1,185.79	-1.31	-42.82
Hedged portfolio				16,796.90	0.00	0.00	-98,540.35	-5,979.70	-4.53	-270.31
Total costs									6,254.55	

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Index option market maker

- Illustration: Drop contracts and re-solve.



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Lesson summary

- Steps in dynamically hedging risk.
 - Identify portfolio's risk exposures.
 - Identify available hedge instruments.
 - Identify lowest cost hedge.
 - Minimize costs.
 - Trading costs
 - Interest cost
 - Erosion of time premium

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