Stock index options

Markets and applications

Stock index options

- Purpose:
  - Describe stock index option markets.
  - Apply index options for managing return/risk.
- Outline:
  - History and status of stock index option markets
  - Insuring portfolios
    - Passive and dynamic
  - Creating protected equity notes as private offerings
  - Creating buffered enhanced notes as private offerings and exchange-traded products

Stock index options

- Index options began trading in March 1983.
  - CBOE introduced options on CBOE 100
    - called “OEX” options
    - index later became “S&P 100”
- Index futures options began in March 1983.
  - CME introduced options on S&P 500 futures

Stock index options

- Contract specifications:
  - European-style, except for OEX options
  - expire Saturday after third Friday
  - exercise price in 5-point increments
  - 100 times index level
  - cash-settlement

Stock index options

Options and Futures Volume by Exchange — February 6, 2020

Reports are available for up to two (2) years. Batch processing information can be found here.

Options

Exchange	Equity		Index/Other		Debt		Exchange Total	
	Volume	Market Share	Volume	Market Share	Volume	Market Share	Volume	Market Share
AMEX	1,901,498	8.42%	0	0.00%	0	0.00%	1,901,498	7.75%
ARCA	1,966,771	8.71%	0	0.00%	0	0.00%	1,966,771	8.02%
BATS	2,101,429	9.30%	836	0.04%	0	0.00%	2,102,265	8.57%
BOX	717,422	3.18%	0	0.00%	0	0.00%	717,422	2.93%
C2	809,935	3.59%	6,289	0.33%	0	0.00%	816,224	3.33%
CBOE	3,247,798	14.38%	1,908,389	98.81%	0	0.00%	5,156,187	21.03%
EDGX	828,137	3.67%	513	0.03%	0	0.00%	828,650	3.38%
EMLD	171,564	0.76%	0	0.00%	0	0.00%	171,564	0.70%
GEM	1,023,713	4.53%	300	0.02%	0	0.00%	1,024,013	4.18%
ISE	1,837,807	8.14%	9,984	0.52%	0	0.00%	1,847,791	7.54%
MCRY	90,808	0.40%	0	0.00%	0	0.00%	90,808	0.37%
MIAX	1,038,088	4.60%	1	0.00%	0	0.00%	1,038,089	4.23%
MPRL	1,171,263	5.18%	0	0.00%	0	0.00%	1,171,263	4.78%
NOBO	51,299	0.23%	0	0.00%	0	0.00%	51,299	0.21%
NSDQ	2,123,738	9.40%	0	0.00%	0	0.00%	2,123,738	8.66%
PHLX	3,508,899	15.53%	5,090	0.26%	0	0.00%	3,513,989	14.33%
OCC Totals	22,590,169	100.00%	1,931,402	100.00%	0	0.00%	24,521,571	100.00%

Stock index futures options

- E-mini S&P 500 contract specifications:
  - American-style
  - expire at open on third Friday together with futures
  - exercise price in 5-point increments
  - 50 times index level
  - delivery contract, except for last day

Stock index futures options

Product name	Index	Volume	Open interest	Turnover
E-mini S&P 500 Options	US	120,706	1,825,680	6.61%
E-mini S&P 500 EOM Options	US	56,081	697,083	8.05%
E-mini S&P 500 Weekly Options - Week 3	US	65,217	676,870	9.64%
E-mini S&P 500 Weekly Options - Week 1	US	141,641	465,180	30.45%
E-mini S&P 500 Weekly Options - Week 2	US	57,727	240,890	23.96%
E-mini S&P 500 Monday Weekly Options - Week 2	US	19,381	73,651	26.31%
S&P 500 Weekly Options - Week 1	US	7,069	53,927	13.11%
S&P 500 Weekly Options - Week 3	US	3,491	52,848	6.61%
S&P 500 Options	US	1,098	50,540	2.17%
E-mini Nasdaq-100 Options	US	7,040	45,326	15.53%
E-mini S&P 500 Wednesday Weekly Options - Week 2	US	21,316	45,320	47.03%
E-mini Nasdaq-100 Weekly Options - Week 3	US	1,810	28,709	6.30%
S&P 500 Weekly Options - Week 2	US	8,234	22,516	36.57%
E-mini S&P 500 Monday Weekly Options - Week 3	US	4,368	21,218	20.59%

Stock index option valuation

- For European-style options written on stock index, reduce index level by present value of promised dividends.

$$PVD = \sum_{i=1}^{n} De^{-r(T-t_i)}$$

$$S^x = S - PVD$$

- Valuation equations are:

Put-call parity: $c - p = S^x - Xe^{-rT}$

BSM call option formula: $c = S^x N(d_1) - Xe^{-rT} N(d_2)$

Stock index futures option valuation

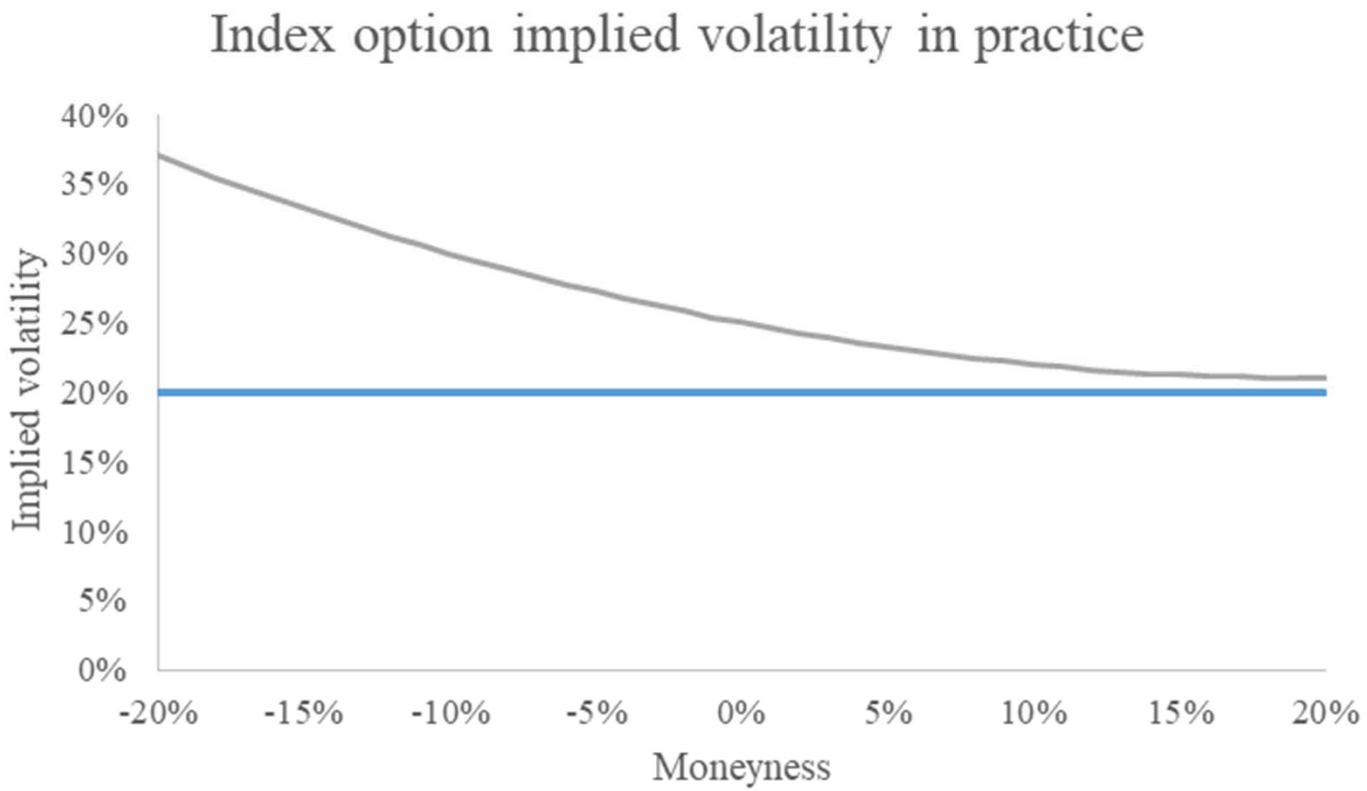
- For European-style options written on stock index futures, valuation equations are:

Put-call parity: $c - p = e^{-rT}\left(F - X\right)$

BSM call option formula: $c = e^{-rT}\left(FN\left(d_1\right) - XN\left(d_2\right)\right)$

S&P 500 index option idiosyncrasies

- Index option implied volatility smirk



Risk management strategies

- Insuring portfolios
  - Passive portfolio insurance
  - Dynamic portfolio insurance
- Creating protected equity notes
- Creating enhanced buffered notes

Portfolio insurance

- *Portfolio insurance* refers to insuring value of portfolio of assets, most commonly stocks.
  - One of most important financial innovations of 1980s.

Portfolio insurance

- Two types of portfolio insurance:
  - *Passive portfolio insurance*: buy index put option while holding stock portfolio
    - index puts began trading in 1983
  - *Dynamic portfolio insurance*: dynamically rebalance portfolio of stocks and T-bills
    - idea conceived by Hayne Leland in 1976
    - LOR had more than $60B under insurance programs by October 1987 market crash

Passive portfolio insurance

- *Passive portfolio insurance* involves buying index put against a position in a stock portfolio. Value of position is

$$V = Se^{-\delta T} + p$$

Passive portfolio insurance

- Cost of insurance (or value of index put) is

$$p = Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

where

$$d_1 = \frac{\ln(S/X) + (r - \delta + .5\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Passive portfolio insurance

- Illustration:
  - Assume index portfolio has
    - value = $50 million
    - volatility rate = 20%
    - dividend yield rate = 1.5%
  - Assume
    - S&P 500 index level = 1,500
    - risk-free rate = 6%
  - Want to insure portfolio value is at least $50 million in one year.
  - Supporting file: Portfolio insurance.xlsx

Passive portfolio insurance

- Illustration:
  - Step 1: Identify number of index units today and at end of one year.
    - Denomination of put is 100.
    - Number of units today is

$$n = \frac{50,000,000}{1,500(100)} = 333.333$$

Passive portfolio insurance

- Illustration:
  - With re-investment of dividend yield, number of index units in one year will be

$$n = 333.33e^{.015(1)} = 338.371$$

  - Need to buy 338.371 index puts.

Passive portfolio insurance

- Illustration:
  - Step 2: Identify exercise price of put.

$$X = \frac{50,000,000}{338.371(100)} = 1,477.67$$

Passive portfolio insurance

- Illustration:
  - Step 3: Identify cost of insurance.
    - Cost of each put is $76.34 per index unit (according to BSM formula).
    - Total cost is $2,583,000.

Passive portfolio insurance

Examine values at expiration.

Portfolio values at expiration			
Index level	Stock portfolio value	Value of put	Insured portfolio value
500	16,918,551	33,081,449	50,000,000
600	20,302,261	29,697,739	50,000,000
700	23,685,972	26,314,028	50,000,000
800	27,069,682	22,930,318	50,000,000
900	30,453,392	19,546,608	50,000,000
1,000	33,837,102	16,162,898	50,000,000
1,100	37,220,812	12,779,188	50,000,000
1,200	40,604,523	9,395,477	50,000,000
1,300	43,988,233	6,011,767	50,000,000
1,400	47,371,943	2,628,057	50,000,000
1,500	50,755,653	0	50,755,653
1,600	54,139,363	0	54,139,363
1,700	57,523,074	0	57,523,074
1,800	60,906,784	0	60,906,784
1,900	64,290,494	0	64,290,494
2,000	67,674,204	0	67,674,204
2,100	71,057,915	0	71,057,915
2,200	74,441,625	0	74,441,625
2,300	77,825,335	0	77,825,335
2,400	81,209,045	0	81,209,045
2,500	84,592,755	0	84,592,755

Passive portfolio insurance

Examine values at expiration.
-- Protected on downside.

Portfolio values at expiration			
Index level	Stock portfolio value	Value of put	Insured portfolio value
500	16,918,551	33,081,449	50,000,000
600	20,302,261	29,697,739	50,000,000
700	23,685,972	26,314,028	50,000,000
800	27,069,682	22,930,318	50,000,000
900	30,453,392	19,546,608	50,000,000
1,000	33,837,102	16,162,898	50,000,000
1,100	37,220,812	12,779,188	50,000,000
1,200	40,604,523	9,395,477	50,000,000
1,300	43,988,233	6,011,767	50,000,000
1,400	47,371,943	2,628,057	50,000,000
1,500	50,755,653	0	50,755,653
1,600	54,139,363	0	54,139,363
1,700	57,523,074	0	57,523,074
1,800	60,906,784	0	60,906,784
1,900	64,290,494	0	64,290,494
2,000	67,674,204	0	67,674,204
2,100	71,057,915	0	71,057,915
2,200	74,441,625	0	74,441,625
2,300	77,825,335	0	77,825,335
2,400	81,209,045	0	81,209,045
2,500	84,592,755	0	84,592,755

Passive portfolio insurance

Examine values at expiration.

-- Retain upside.

Portfolio values at expiration			
Index level	Stock portfolio value	Value of put	Insured portfolio value
500	16,918,551	33,081,449	50,000,000
600	20,302,261	29,697,739	50,000,000
700	23,685,972	26,314,028	50,000,000
800	27,069,682	22,930,318	50,000,000
900	30,453,392	19,546,608	50,000,000
1,000	33,837,102	16,162,898	50,000,000
1,100	37,220,812	12,779,188	50,000,000
1,200	40,604,523	9,395,477	50,000,000
1,300	43,988,233	6,011,767	50,000,000
1,400	47,371,943	2,628,057	50,000,000
1,500	50,755,653	0	50,755,653
1,600	54,139,363	0	54,139,363
1,700	57,523,074	0	57,523,074
1,800	60,906,784	0	60,906,784
1,900	64,290,494	0	64,290,494
2,000	67,674,204	0	67,674,204
2,100	71,057,915	0	71,057,915
2,200	74,441,625	0	74,441,625
2,300	77,825,335	0	77,825,335
2,400	81,209,045	0	81,209,045
2,500	84,592,755	0	84,592,755

Passive portfolio insurance

Examine initial values.

Portfolio values with one year to expiration			
Index level	Stock portfolio value	Value of put	Insured portfolio value
500	16,666,667	30,421,560	47,088,227
600	20,000,000	27,088,239	47,088,239
700	23,333,333	23,755,263	47,088,596
800	26,666,667	20,426,203	47,092,869
900	30,000,000	17,119,553	47,119,553
1,000	33,333,333	13,889,916	47,223,250
1,100	36,666,667	10,839,846	47,506,513
1,200	40,000,000	8,100,014	48,100,014
1,300	43,333,333	5,783,980	49,117,313
1,400	46,666,667	3,948,601	50,615,267
1,500	50,000,000	2,583,000	52,583,000
1,600	53,333,333	1,624,612	54,957,946
1,700	56,666,667	986,331	57,652,998
1,800	60,000,000	580,381	60,580,381
1,900	63,333,333	332,300	63,665,633
2,000	66,666,667	185,807	66,852,474
2,100	70,000,000	101,801	70,101,801
2,200	73,333,333	54,814	73,388,147
2,300	76,666,667	29,082	76,695,749
2,400	80,000,000	15,240	80,015,240
2,500	83,333,333	7,904	83,341,237

Passive portfolio insurance

Examine initial values.

If index level falls to 500 immediately, portfolio value is 47,088,227.

Portfolio values with one year to expiration			
Index level	Stock portfolio value	Value of put	Insured portfolio value
500	16,666,667	30,421,560	47,088,227
600	20,000,000	27,088,239	47,088,239
700	23,333,333	23,755,263	47,088,596
800	26,666,667	20,426,203	47,092,869
900	30,000,000	17,119,553	47,119,553
1,000	33,333,333	13,889,916	47,223,250
1,100	36,666,667	10,839,846	47,506,513
1,200	40,000,000	8,100,014	48,100,014
1,300	43,333,333	5,783,980	49,117,313
1,400	46,666,667	3,948,601	50,615,267
1,500	50,000,000	2,583,000	52,583,000
1,600	53,333,333	1,624,612	54,957,946
1,700	56,666,667	986,331	57,652,998
1,800	60,000,000	580,381	60,580,381
1,900	63,333,333	332,300	63,665,633
2,000	66,666,667	185,807	66,852,474
2,100	70,000,000	101,801	70,101,801
2,200	73,333,333	54,814	73,388,147
2,300	76,666,667	29,082	76,695,749
2,400	80,000,000	15,240	80,015,240
2,500	83,333,333	7,904	83,341,237

Passive portfolio insurance

Examine initial values.

If index level falls to 500 immediately, portfolio value is 47,088,227.

$$\$47{,}088{,}238=\$50{,}000{,}000e^{-.06(1)}$$

Portfolio values with one year to expiration

Index level	Stock portfolio value	Value of put	Insured portfolio value
500	16,666,667	30,421,560	47,088,227
600	20,000,000	27,088,239	47,088,239
700	23,333,333	23,755,263	47,088,596
800	26,666,667	20,426,203	47,092,869
900	30,000,000	17,119,553	47,119,553
1,000	33,333,333	13,889,916	47,223,250
1,100	36,666,667	10,839,846	47,506,513
1,200	40,000,000	8,100,014	48,100,014
1,300	43,333,333	5,783,980	49,117,313
1,400	46,666,667	3,948,601	50,615,267
1,500	50,000,000	2,583,000	52,583,000
1,600	53,333,333	1,624,612	54,957,946
1,700	56,666,667	986,331	57,652,998
1,800	60,000,000	580,381	60,580,381
1,900	63,333,333	332,300	63,665,633
2,000	66,666,667	185,807	66,852,474
2,100	70,000,000	101,801	70,101,801
2,200	73,333,333	54,814	73,388,147
2,300	76,666,667	29,082	76,695,749
2,400	80,000,000	15,240	80,015,240
2,500	83,333,333	7,904	83,341,237

Dynamic portfolio insurance

- *Dynamic portfolio insurance* does not involve buying insurance directly.
  - Dynamically insure using stock portfolio and T-bills.
    - As market rises, sell T-bills and buy stocks.
    - As market falls, sell stocks and buy T-bills.

Dynamic portfolio insurance

- To identify portfolio weights for stocks and T-bills, solve:
  - *Value constraint*

$$V = Se^{-\delta T} + p = w_S Se^{-\delta T} + w_B Xe^{-rT}$$

  - *Change in value or delta constraint*

$$\frac{\partial V}{\partial S} = e^{-\delta T} - e^{-\delta T} N(-d_1) = w_S e^{-\delta T} + w_B \frac{\partial Xe^{-rT}}{\partial S} = w_S e^{-\delta T}$$

Dynamic portfolio insurance

- To identify portfolio weights for stocks and T-bills, solve:

$$w_S = N(d_1) \text{ and } w_B = N(-d_2)$$

Dynamic portfolio insurance

Examine initial values.

Portfolio values with one year to expiration					
Index level	Stock portfolio value	Value bonds	w_S	w_B	Insured portfolio value
500	16,666,667	47,088,227	0.0000	1.0000	47,088,227
600	20,000,000	47,088,227	0.0000	1.0000	47,088,239
700	23,333,333	47,088,227	0.0003	0.9998	47,088,596
800	26,666,667	47,088,227	0.0030	0.9984	47,092,869
900	30,000,000	47,088,227	0.0156	0.9907	47,119,553
1,000	33,333,333	47,088,227	0.0518	0.9662	47,223,250
1,100	36,666,667	47,088,227	0.1249	0.9116	47,506,513
1,200	40,000,000	47,088,227	0.2371	0.8201	48,100,014
1,300	43,333,333	47,088,227	0.3762	0.6969	49,117,313
1,400	46,666,667	47,088,227	0.5219	0.5576	50,615,267
1,500	50,000,000	47,088,227	0.6554	0.4207	52,583,000
1,600	53,333,333	47,088,227	0.7651	0.3006	54,957,946
1,700	56,666,667	47,088,227	0.8475	0.2045	57,652,998
1,800	60,000,000	47,088,227	0.9052	0.1332	60,580,381
1,900	63,333,333	47,088,227	0.9432	0.0835	63,665,633
2,000	66,666,667	47,088,227	0.9670	0.0507	66,852,474
2,100	70,000,000	47,088,227	0.9813	0.0299	70,101,801
2,200	73,333,333	47,088,227	0.9897	0.0172	73,388,147
2,300	76,666,667	47,088,227	0.9944	0.0097	76,695,749
2,400	80,000,000	47,088,227	0.9970	0.0054	80,015,240
2,500	83,333,333	47,088,227	0.9984	0.0029	83,341,237

Dynamic portfolio insurance

Examine initial values.

As stock prices fall, sell stocks and buy T-bills.

Portfolio values with one year to expiration					
Index level	Stock portfolio value	Value bonds	w_S	w_B	Insured portfolio value
500	16,666,667	47,088,227	0.0000	1.0000	47,088,227
600	20,000,000	47,088,227	0.0000	1.0000	47,088,239
700	23,333,333	47,088,227	0.0003	0.9998	47,088,596
800	26,666,667	47,088,227	0.0030	0.9984	47,092,869
900	30,000,000	47,088,227	0.0156	0.9907	47,119,553
1,000	33,333,333	47,088,227	0.0518	0.9662	47,223,250
1,100	36,666,667	47,088,227	0.1249	0.9116	47,506,513
1,200	40,000,000	47,088,227	0.2371	0.8201	48,100,014
1,300	43,333,333	47,088,227	0.3762	0.6969	49,117,313
1,400	46,666,667	47,088,227	0.5219	0.5576	50,615,267
1,500	50,000,000	47,088,227	0.6554	0.4207	52,583,000
1,600	53,333,333	47,088,227	0.7651	0.3006	54,957,946
1,700	56,666,667	47,088,227	0.8475	0.2045	57,652,998
1,800	60,000,000	47,088,227	0.9052	0.1332	60,580,381
1,900	63,333,333	47,088,227	0.9432	0.0835	63,665,633
2,000	66,666,667	47,088,227	0.9670	0.0507	66,852,474
2,100	70,000,000	47,088,227	0.9813	0.0299	70,101,801
2,200	73,333,333	47,088,227	0.9897	0.0172	73,388,147
2,300	76,666,667	47,088,227	0.9944	0.0097	76,695,749
2,400	80,000,000	47,088,227	0.9970	0.0054	80,015,240
2,500	83,333,333	47,088,227	0.9984	0.0029	83,341,237

Dynamic portfolio insurance

Examine initial values.

As stock prices rise, sell T-bills and buy stocks.

Portfolio values with one year to expiration					
Index level	Stock portfolio value	Value bonds	w_S	w_B	Insured portfolio value
500	16,666,667	47,088,227	0.0000	1.0000	47,088,227
600	20,000,000	47,088,227	0.0000	1.0000	47,088,239
700	23,333,333	47,088,227	0.0003	0.9998	47,088,596
800	26,666,667	47,088,227	0.0030	0.9984	47,092,869
900	30,000,000	47,088,227	0.0156	0.9907	47,119,553
1,000	33,333,333	47,088,227	0.0518	0.9662	47,223,250
1,100	36,666,667	47,088,227	0.1249	0.9116	47,506,513
1,200	40,000,000	47,088,227	0.2371	0.8201	48,100,014
1,300	43,333,333	47,088,227	0.3762	0.6969	49,117,313
1,400	46,666,667	47,088,227	0.5219	0.5576	50,615,267
1,500	50,000,000	47,088,227	0.6554	0.4207	52,583,000
1,600	53,333,333	47,088,227	0.7651	0.3006	54,957,946
1,700	56,666,667	47,088,227	0.8475	0.2045	57,652,998
1,800	60,000,000	47,088,227	0.9052	0.1332	60,580,381
1,900	63,333,333	47,088,227	0.9432	0.0835	63,665,633
2,000	66,666,667	47,088,227	0.9670	0.0507	66,852,474
2,100	70,000,000	47,088,227	0.9813	0.0299	70,101,801
2,200	73,333,333	47,088,227	0.9897	0.0172	73,388,147
2,300	76,666,667	47,088,227	0.9944	0.0097	76,695,749
2,400	80,000,000	47,088,227	0.9970	0.0054	80,015,240
2,500	83,333,333	47,088,227	0.9984	0.0029	83,341,237

Dynamic portfolio insurance

Examine initial values.

Same insured portfolio values as passive insurance.

Portfolio values with one year to expiration					
Index level	Stock portfolio value	Value bonds	w_S	w_B	Insured portfolio value
500	16,666,667	47,088,227	0.0000	1.0000	47,088,227
600	20,000,000	47,088,227	0.0000	1.0000	47,088,239
700	23,333,333	47,088,227	0.0003	0.9998	47,088,596
800	26,666,667	47,088,227	0.0030	0.9984	47,092,869
900	30,000,000	47,088,227	0.0156	0.9907	47,119,553
1,000	33,333,333	47,088,227	0.0518	0.9662	47,223,250
1,100	36,666,667	47,088,227	0.1249	0.9116	47,506,513
1,200	40,000,000	47,088,227	0.2371	0.8201	48,100,014
1,300	43,333,333	47,088,227	0.3762	0.6969	49,117,313
1,400	46,666,667	47,088,227	0.5219	0.5576	50,615,267
1,500	50,000,000	47,088,227	0.6554	0.4207	52,583,000
1,600	53,333,333	47,088,227	0.7651	0.3006	54,957,946
1,700	56,666,667	47,088,227	0.8475	0.2045	57,652,998
1,800	60,000,000	47,088,227	0.9052	0.1332	60,580,381
1,900	63,333,333	47,088,227	0.9432	0.0835	63,665,633
2,000	66,666,667	47,088,227	0.9670	0.0507	66,852,474
2,100	70,000,000	47,088,227	0.9813	0.0299	70,101,801
2,200	73,333,333	47,088,227	0.9897	0.0172	73,388,147
2,300	76,666,667	47,088,227	0.9944	0.0097	76,695,749
2,400	80,000,000	47,088,227	0.9970	0.0054	80,015,240
2,500	83,333,333	47,088,227	0.9984	0.0029	83,341,237

Dynamic portfolio insurance

- *Dynamic portfolio insurance* does not involve buying insurance directly.
  - Problem: Stock portfolio managers may be reluctant to sell stocks.
  - Solution: Synthetically sell stocks using index futures, which had been recently launched.
    - Replicating portfolio is not unique.

Dynamic portfolio insurance

- *Dynamic portfolio insurance* does not involve buying insurance directly.
  - Buy and hold stock portfolio.
  - Dynamically insure using index futures/T-bills.
    - As market rises, buy futures and sell T-bills.
    - As market falls, sell futures and buy T-bills.

Dynamic portfolio insurance

- To identify portfolio weights, solve.
  - *Value constraint*

$$V = Se^{-\delta T} + p = Se^{-\delta T} + w_F F + w_B Xe^{-rT}$$
$$= Se^{-\delta T} + w_B Xe^{-rT}$$

  - *Change in value or delta constraint*

$$\frac{\partial V}{\partial S} = e^{-\delta T} - e^{-\delta T} N(-d_1) = e^{-\delta T} + w_F \frac{\partial F}{\partial S}$$
$$= e^{-\delta T} + w_F e^{(r-\delta)T}$$

Dynamic portfolio insurance

- Weights for stocks and futures are:

$$w_B = \frac{p}{Xe^{-rT}}$$

$$w_F = -e^{-rT} N\left(-d_1\right)$$

Dynamic portfolio insurance

As stock prices fall, sell more futures and buy more T-bills.

Portfolio values with one year to expiration						
Index level	Futures price	Stock portfolio value	Bond value	w_B	w_F	Insured portfolio value
500	523.01	16,666,667	47,088,227	0.6461	-0.9418	47,088,227
600	627.62	20,000,000	47,088,227	0.5753	-0.9418	47,088,239
700	732.22	23,333,333	47,088,227	0.5045	-0.9415	47,088,596
800	836.82	26,666,667	47,088,227	0.4338	-0.9389	47,092,869
900	941.43	30,000,000	47,088,227	0.3636	-0.9271	47,119,553
1,000	1046.03	33,333,333	47,088,227	0.2950	-0.8929	47,223,250
1,100	1150.63	36,666,667	47,088,227	0.2302	-0.8241	47,506,513
1,200	1255.23	40,000,000	47,088,227	0.1720	-0.7185	48,100,014
1,300	1359.84	43,333,333	47,088,227	0.1228	-0.5875	49,117,313
1,400	1464.44	46,666,667	47,088,227	0.0839	-0.4502	50,615,267
1,500	1569.04	50,000,000	47,088,227	0.0549	-0.3245	52,583,000
1,600	1673.64	53,333,333	47,088,227	0.0345	-0.2213	54,957,946
1,700	1778.25	56,666,667	47,088,227	0.0209	-0.1436	57,652,998
1,800	1882.85	60,000,000	47,088,227	0.0123	-0.0893	60,580,381
1,900	1987.45	63,333,333	47,088,227	0.0071	-0.0535	63,665,633
2,000	2092.06	66,666,667	47,088,227	0.0039	-0.0311	66,852,474
2,100	2196.66	70,000,000	47,088,227	0.0022	-0.0176	70,101,801
2,200	2301.26	73,333,333	47,088,227	0.0012	-0.0097	73,388,147
2,300	2405.86	76,666,667	47,088,227	0.0006	-0.0053	76,695,749
2,400	2510.47	80,000,000	47,088,227	0.0003	-0.0028	80,015,240
2,500	2615.07	83,333,333	47,088,227	0.0002	-0.0015	83,341,237

Dynamic portfolio insurance

Gains from futures are used to buy more T-bills.

Portfolio values with one year to expiration						
Index level	Futures price	Stock portfolio value	Bond value	w_B	w_F	Insured portfolio value
500	523.01	16,666,667	47,088,227	0.6461	-0.9418	47,088,227
600	627.62	20,000,000	47,088,227	0.5753	-0.9418	47,088,239
700	732.22	23,333,333	47,088,227	0.5045	-0.9415	47,088,596
800	836.82	26,666,667	47,088,227	0.4338	-0.9389	47,092,869
900	941.43	30,000,000	47,088,227	0.3636	-0.9271	47,119,553
1,000	1046.03	33,333,333	47,088,227	0.2950	-0.8929	47,223,250
1,100	1150.63	36,666,667	47,088,227	0.2302	-0.8241	47,506,513
1,200	1255.23	40,000,000	47,088,227	0.1720	-0.7185	48,100,014
1,300	1359.84	43,333,333	47,088,227	0.1228	-0.5875	49,117,313
1,400	1464.44	46,666,667	47,088,227	0.0839	-0.4502	50,615,267
1,500	1569.04	50,000,000	47,088,227	0.0549	-0.3245	52,583,000
1,600	1673.64	53,333,333	47,088,227	0.0345	-0.2213	54,957,946
1,700	1778.25	56,666,667	47,088,227	0.0209	-0.1436	57,652,998
1,800	1882.85	60,000,000	47,088,227	0.0123	-0.0893	60,580,381
1,900	1987.45	63,333,333	47,088,227	0.0071	-0.0535	63,665,633
2,000	2092.06	66,666,667	47,088,227	0.0039	-0.0311	66,852,474
2,100	2196.66	70,000,000	47,088,227	0.0022	-0.0176	70,101,801
2,200	2301.26	73,333,333	47,088,227	0.0012	-0.0097	73,388,147
2,300	2405.86	76,666,667	47,088,227	0.0006	-0.0053	76,695,749
2,400	2510.47	80,000,000	47,088,227	0.0003	-0.0028	80,015,240
2,500	2615.07	83,333,333	47,088,227	0.0002	-0.0015	83,341,237

Dynamic portfolio insurance

As stock prices rise, futures losses are covered using T-bills.

Portfolio values with one year to expiration						
Index level	Futures price	Stock portfolio value	Bond value	w_B	w_F	Insured portfolio value
500	523.01	16,666,667	47,088,227	0.6461	-0.9418	47,088,227
600	627.62	20,000,000	47,088,227	0.5753	-0.9418	47,088,239
700	732.22	23,333,333	47,088,227	0.5045	-0.9415	47,088,596
800	836.82	26,666,667	47,088,227	0.4338	-0.9389	47,092,869
900	941.43	30,000,000	47,088,227	0.3636	-0.9271	47,119,553
1,000	1046.03	33,333,333	47,088,227	0.2950	-0.8929	47,223,250
1,100	1150.63	36,666,667	47,088,227	0.2302	-0.8241	47,506,513
1,200	1255.23	40,000,000	47,088,227	0.1720	-0.7185	48,100,014
1,300	1359.84	43,333,333	47,088,227	0.1228	-0.5875	49,117,313
1,400	1464.44	46,666,667	47,088,227	0.0839	-0.4502	50,615,267
1,500	1569.04	50,000,000	47,088,227	0.0549	-0.3245	52,583,000
1,600	1673.64	53,333,333	47,088,227	0.0345	-0.2213	54,957,946
1,700	1778.25	56,666,667	47,088,227	0.0209	-0.1436	57,652,998
1,800	1882.85	60,000,000	47,088,227	0.0123	-0.0893	60,580,381
1,900	1987.45	63,333,333	47,088,227	0.0071	-0.0535	63,665,633
2,000	2092.06	66,666,667	47,088,227	0.0039	-0.0311	66,852,474
2,100	2196.66	70,000,000	47,088,227	0.0022	-0.0176	70,101,801
2,200	2301.26	73,333,333	47,088,227	0.0012	-0.0097	73,388,147
2,300	2405.86	76,666,667	47,088,227	0.0006	-0.0053	76,695,749
2,400	2510.47	80,000,000	47,088,227	0.0003	-0.0028	80,015,240
2,500	2615.07	83,333,333	47,088,227	0.0002	-0.0015	83,341,237

Creating protected equity notes

- *Protected equity note* (PEN) is contract that provides guaranteed minimum rate of return on investment plus proportion of price appreciation or total return in underlying stock/index.
  - Also called:
    - principal protected notes
    - equity-linked certificates of deposit
    - bull certificates of deposit

Protected equity notes

- Notation:

  $V \equiv$ principal invested in PEN

  $S \equiv$ index level scaled such that $S = V$

  $g \equiv$ guaranteed investment return on principal

  $k \equiv$ participation rate (e.g., proportion of index gain if market rises)

  - All other notation is as before.

Protected equity notes

- Holding PEN is like holding risk-free bonds plus call option.
  - T-bills provide guaranteed minimum principal
  - call provides upside

Protected equity notes

- Guaranteed minimum principal of PEN

$$Ve^{gT}$$

- Amount of T-bills to buy today to provide for guarantee is

$$Ve^{gT}e^{-rT} = Ve^{(g-r)T}$$

Protected equity notes

- Equity share is determined by call with exercise price equal to guaranteed principal, Ve^{gT}.

$$c = Se^{-\delta T} N(d_1) - Ve^{gT} e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(Se^{-\delta T} / Ve^{(g-r)T}) + .5\sigma^2 T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Protected equity notes

- BSM formula, however, implies 100% participation rate. Bank states *k*%. Value of PEN is, therefore,

$$\text{PEN} = Ve^{(g-r)T} + kc$$

Protected equity notes

- Illustration:
  - Want invest $100,000 in protected equity note on price appreciation of S&P 500 index for one year.
    - Bank offers guarantee return on principal or 2% plus 30% of price appreciation in S&P 500 index.
    - S&P 500 has:
      - Dividend yield of 1.5%
      - Volatility rate of 30%
    - Risk-free interest rate is 6%.
    - Supporting file: Protected equity note.xlsx

Protected equity notes

- Guaranteed minimum principal of PEN is

$$Ve^{gT} = 100,000 \times e^{.02(1)} = 102,020.13$$

- Present value of risk-free bonds that protect principal is

$$Ve^{(g-r)T} = e^{-.06(1)} \times 102,020.13 = 96,078.94$$

Protected equity notes

- Value of call providing 100% participation is

$$c = 100,000e^{-.015(1)}N\left(d_1\right) - 102,020.13e^{-.06(1)}N\left(d_2\right) = 7,495.32$$

where

$$d_1 = \frac{\ln\left(100,000e^{-.015(1)} / 102,020.13e^{-.06(1)}\right) + .5\left(.16^2\right)1}{.16\sqrt{1}} = .2363,$$

$$d_2 = d_1 - .16\sqrt{1} = .0763$$

Protected equity notes

- Cost of what bank provides is, therefore,

$$\text{PEN} = Ve^{(g-r)T} + kc$$

$$= 96,078.94 + .3\left(7,495.32\right) = 98,327.54$$

- Bank's margin is

$$\text{Margin on PEN} = 100,000 - 98,327.54 = 1,672.46$$

Protected equity notes

- Bank offered 30% of principal. If they offered 30% of total return, set dividend yield rate to 0% and use

$$c = SN(d_1) - Ve^{(g-r)T} N(d_2)$$

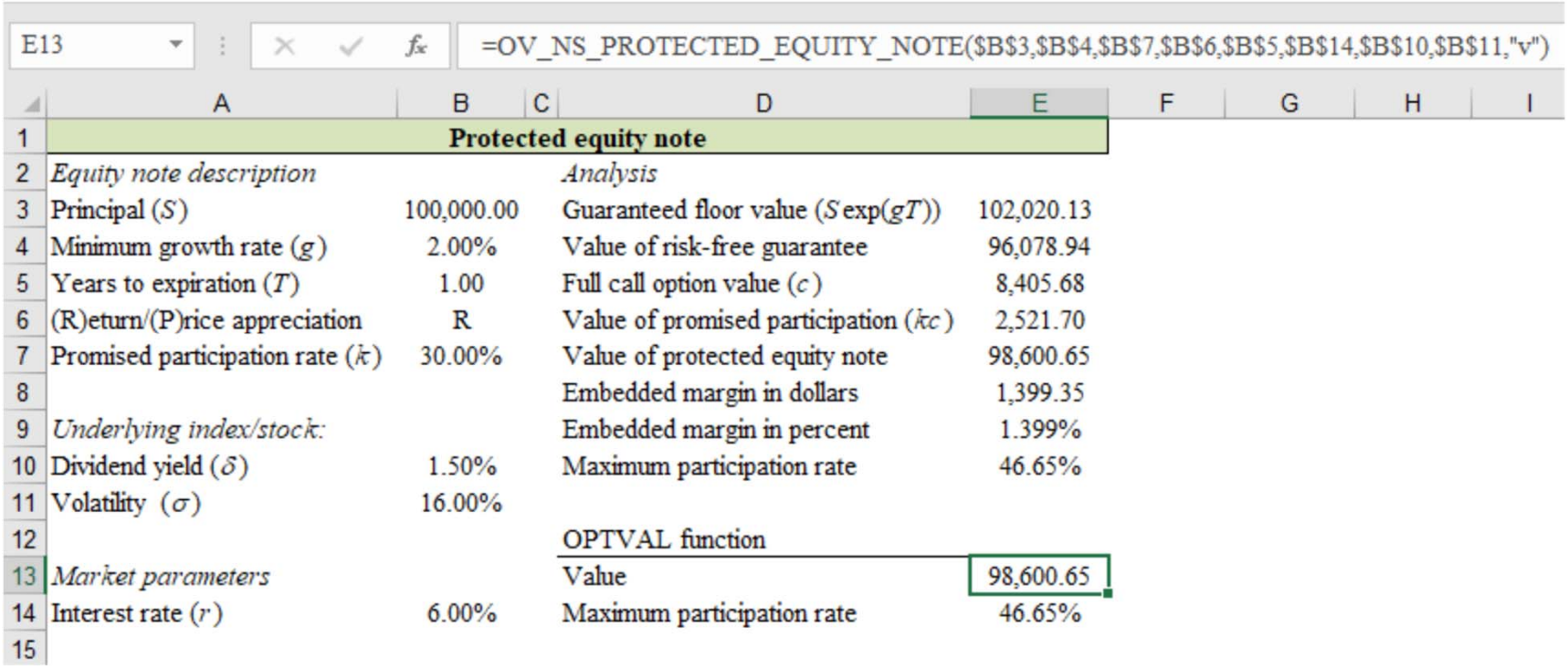
where

$$d_1 = \frac{\ln(S / Ve^{(g-r)T}) + .5\sigma^2 T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Protected equity notes

- OPTVAL function for valuing protected equity notes.
  - Supporting file: Protected equity notes.xlsx



E13 =OV_NS_PROTECTED_EQUITY_NOTE(B3,B4,B7,B6,B5,B14,B10,B11,"v")

	A	B	C	D	E	F	G	H	I
1	**Protected equity note**								
2	*Equity note description*			*Analysis*					
3	Principal (S)	100,000.00		Guaranteed floor value ($S\exp(gT)$)	102,020.13				
4	Minimum growth rate (g)	2.00%		Value of risk-free guarantee	96,078.94				
5	Years to expiration (T)	1.00		Full call option value (c)	8,405.68				
6	(R)eturn/(P)rice appreciation	R		Value of promised participation (kc)	2,521.70				
7	Promised participation rate (k)	30.00%		Value of protected equity note	98,600.65				
8				Embedded margin in dollars	1,399.35				
9	*Underlying index/stock:*			Embedded margin in percent	1.399%				
10	Dividend yield (δ)	1.50%		Maximum participation rate	46.65%				
11	Volatility (σ)	16.00%							
12				OPTVAL function					
13	*Market parameters*			Value	98,600.65				
14	Interest rate (r)	6.00%		Maximum participation rate	46.65%				
15									

Protected equity notes

- How does bank hedge?
  - Can hedge passively by buying index call options. (cheapest)
  - Can hedge dynamically.

Buffered return enhanced notes

- Protected equity notes evolved into more complicated products.
- Banks now offer high-wealth individuals “special” products with more complicated but “attractive” payoff structures.
  - Buffered enhanced return notes is one example.
- ETF issuers came in and undercut margins.
  - Innovator ETFs

Buffered return enhanced notes

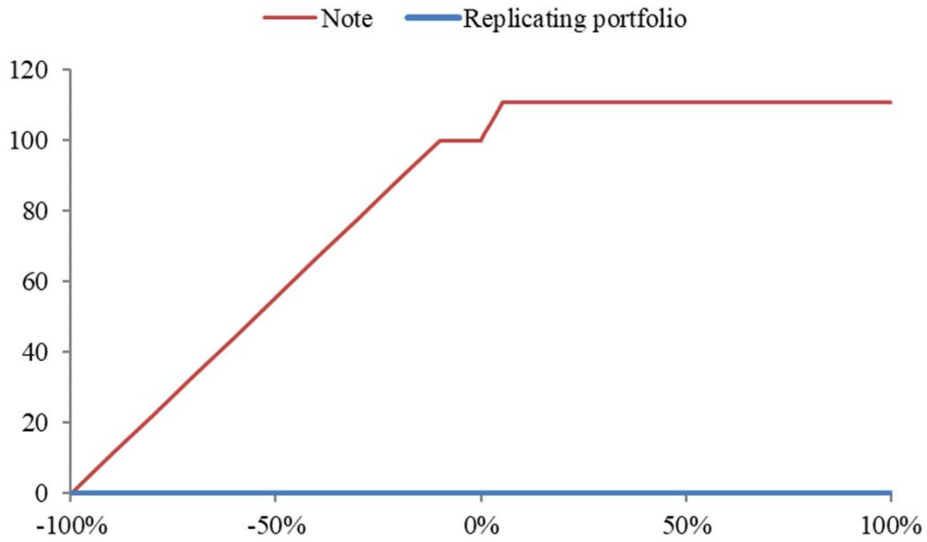
- HSBC Buffered Enhanced Notes
  - Note description: HSBC buffered return notes.pdf
  - Supporting file: HSBC buffered return notes.xlsx

HSBC notes

- HSBC Buffered Enhanced Notes terms
  - Reference asset: S&P 500 index
  - Upside participation rate: 200%
  - Maximum return: 10.74%
  - Buffer amount: 10.00%
  - Downside leverage factor: 1.11111

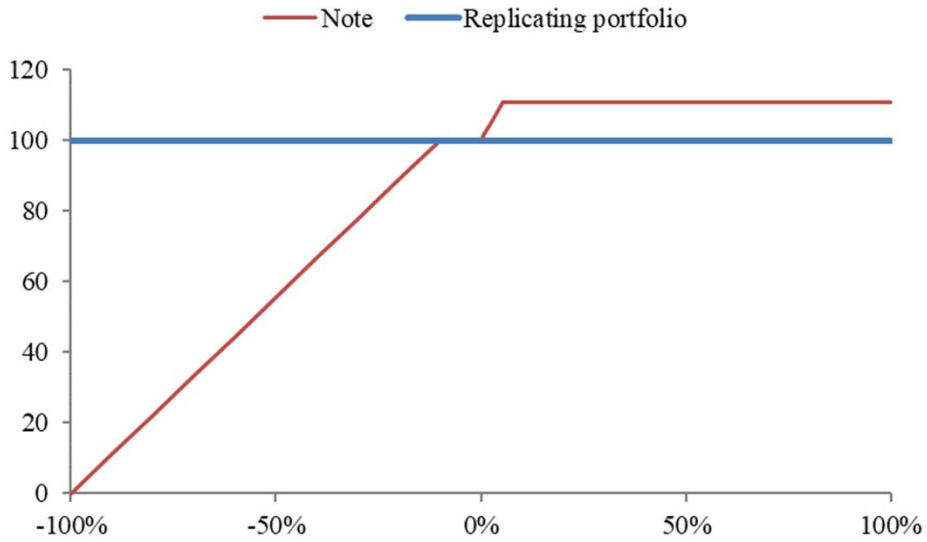
HSBC notes

- HSBC Buffered Enhanced Notes payoffs
  - Want to mimic payoffs using exchange-traded options.



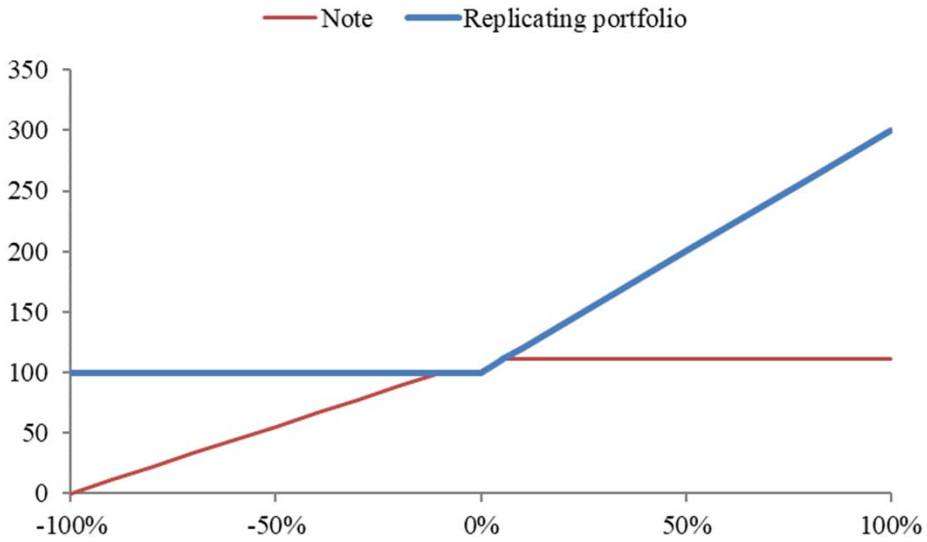
HSBC notes

- HSBC Buffered Enhanced Notes payoffs
  - Buy T-bills with face value of 100.



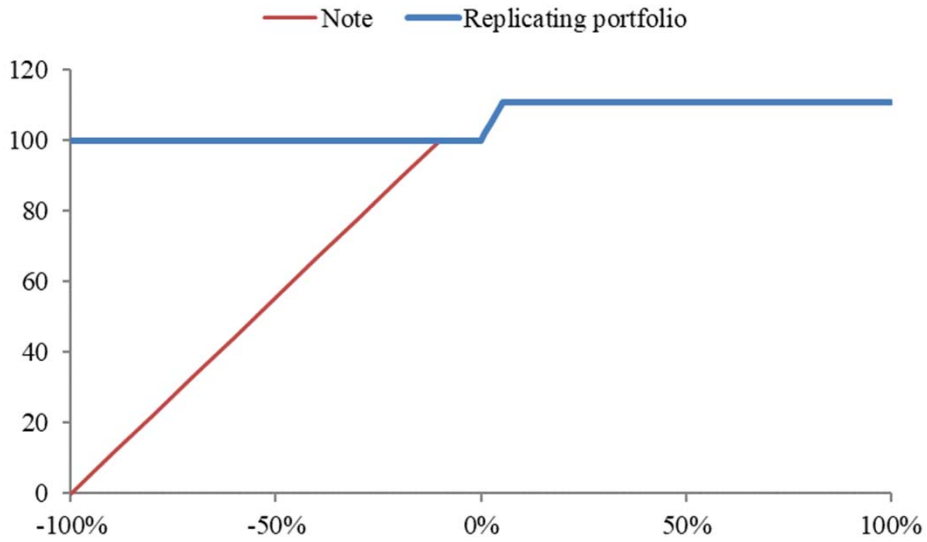
HSBC notes

- HSBC Buffered Enhanced Notes payoffs
  - Buy 2 ATM calls.



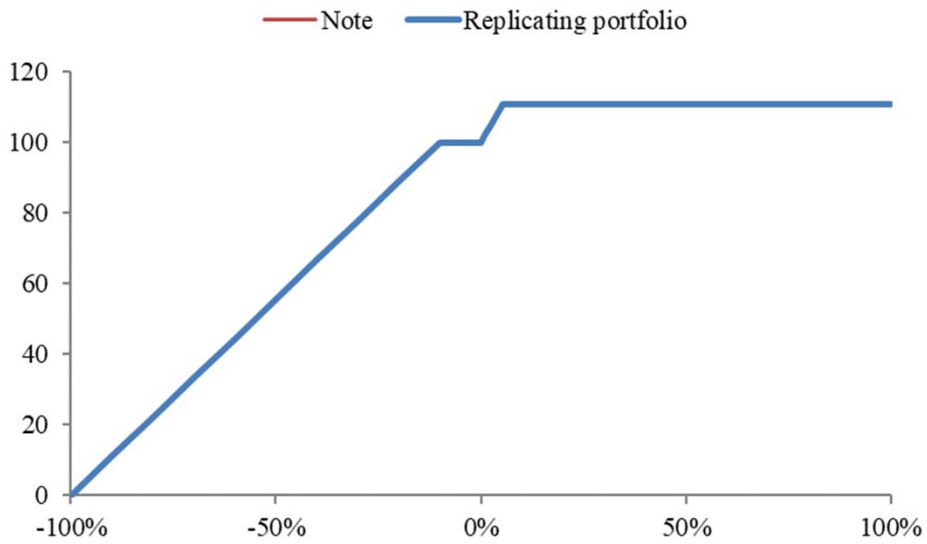
HSBC notes

- HSBC Buffered Enhanced Notes payoffs
  - Sell 2 calls with exercise price 5.37.



HSBC notes

- HSBC Buffered Enhanced Notes payoffs
  - Sell 1.11111 puts with exercise price 90.00.



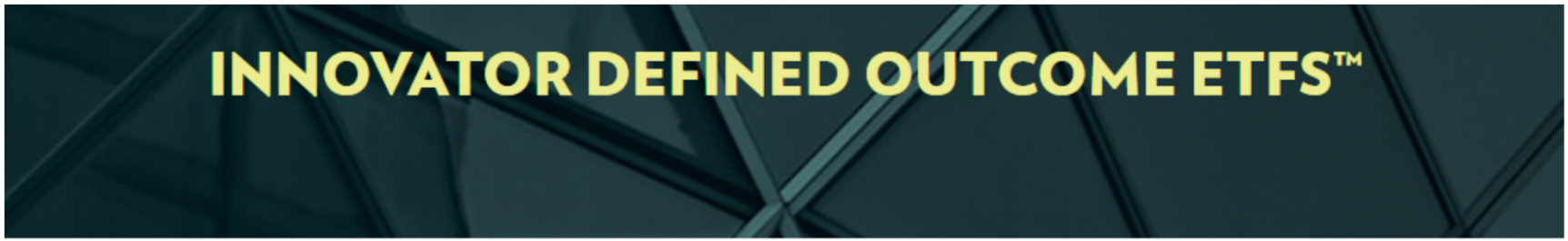
HSBC notes

- HSBC Buffered Enhanced Notes value

Buffered return enhanced note valuation					
Note description		*Valuation*	*No. of options*	*Exercise price*	*Total value*
Principal (S)	100.00				
Cap on index	5.370%	Buy ATM calls.	2	100.00	19.070
Upside leverage factor	2	Sell OTM calls.	-2	105.37	-14.610
Maximum return	10.740%	Sell OTM puts.	-1.1111	90.00	-4.671
Buffered amount	10.000%	Buy risk-free bonds			97.045
Downside leverage factor	1.1111	Value of buffered return enhanced note			96.833
Years to expiration (T)	1.00				
(R)eturn/(P)rice appreciation	P	OptVal function			
		Value of buffered return enhanced note			96.833
Underlying index/stock:					
Dividend yield (δ)	1.90%	Margin			3.167
Volatility (σ)	23.14%	Margin (%)			3.27%
Market parameters					
Interest rate (r)	3.00%				

HSBC notes

- ETF industry offers same products at lower cost.
  - http://www.innovatoretfs.com/



Defined Outcome Product Table

As of 2/7/2020

Download table data (CSV)

TICKER	BUFFER TYPE	SERIES	FUND PRICE	FUND RETURN	INDEX	INDEX RETURN	REMAINING CAP	REMAINING BUFFER	DOWNSIDE BEFORE BUFFER	REMAINING OUTCOME PERIOD	MATERIALS
BFEB›	S&P 500 Buffer ETF	February	$25.05	2.14%	SPX	3.23%	10.63%	9.75%	-2.10%	359 days	Fact Sheet | Prospectus
PFEB›	S&P 500 Power Buffer ETF	February	$24.89	1.49%	SPX	3.23%	6.92%	16.19%	-1.47%	359 days	Fact Sheet | Prospectus
UFEB›	S&P 500 Ultra Buffer ETF	February	$24.79	1.08%	SPX	3.23%	6.44%	31.02%	-6.02%	359 days	Fact Sheet | Prospectus
IJAN›	MSCI EAFE Power Buffer ETF	January	$24.50	0.23%	MXEA	-0.42%	9.83%	14.42%	-0.22%	328 days	Fact Sheet | Prospectus
EJAN›	MSCI EM Power Buffer ETF	January	$26.59	-0.78%	MXEF	-2.40%	13.67%	13.70%	0.00%	328 days	Fact Sheet | Prospectus

HSBC notes

- ETF industry offers same products at lower cost.
  - http://www.innovatoretfs.com/

DETAILS

Ticker	BFEB
Listing date	February 3, 2020
Exposure	S&P 500 Price Index[1]
Starting cap	13.00%
Starting buffer	9.00%
Outcome period	2/1/2020 - 1/31/2021
Rebalance frequency	Annual
Expense ratio	0.79%
Net assets	-
Exchange	Cboe BZX
Series	February

KEY POINTS

- Defined downside buffer levels
- Exposure to S&P 500 upside performance (to a cap)
- Low cost, flexible, liquid, and transparent
- No credit risk
- Rebalances annually and can be held indefinitely

PAYOFF PROFILE (1 YEAR)

S&P 500 Price Index[1]
Innovator S&P 500 Buffer ETF

For information purposes only. Does not represent actual fund performance. Intended to illustrate the return profile the investment objective seeks to achieve relative to the S&P 500. Illustration does not account for fund fees and expenses.

HSBC notes

- ETF industry offers same products at lower cost.
  - http://www.innovatoretfs.com/

DETAILS

Ticker	BFEB
Listing date	February 3, 2020
Exposure	S&P 500 Price Index[1]
Starting cap	13.00%
Starting buffer	9.00%
Outcome period	2/1/2020 - 1/31/2021
Rebalance frequency	Annual
Expense ratio	0.79%
Net assets	-
Exchange	Cboe BZX
Series	February

KEY POINTS

- Defined downside buffer levels
- Exposure to S&P 500 upside performance (to a cap)
- Low cost, flexible, liquid, and transparent
- No credit risk
- Rebalances annually and can be held indefinitely

PAYOFF PROFILE (1 YEAR)

S&P 500 Price Index[1]
Innovator S&P 500 Buffer ETF

For information purposes only. Does not represent actual fund performance. Intended to illustrate the return profile the investment objective seeks to achieve relative to the S&P 500. Illustration does not account for fund fees and expenses.

Lesson summary

- Dominant markets are S&P 500 option markets, same as futures.
- Applications of stock index options
  - Insuring portfolios
    - Passive and dynamic
  - Creating protected equity notes
  - Creating buffered return enhanced notes
    - From private OTC notes to exchange-traded products