Stock index futures

Markets and applications

Stock index derivatives

- Purpose:
  - Describe stock index derivatives markets.
  - Apply index derivatives for managing return/risk.
- Outline:
  - History of stock index products
  - Stock index construction
  - Index arbitrage
  - Hedging stock market risk

Stock index futures

- History of listings in US
  - Value Line futures
    - Kansas City Board of Trade in February 1982
  - S&P 500 futures
    - Chicago Mercantile Exchange in April 1982
  - Major Market Index futures
    - Chicago Board of Trade in July 1984
  - DJIA futures
    - Chicago Board of Trade in October 1997

Stock index futures

- History of listings in other countries
  - All Ordinaries
    - SFE in February 1983
  - TSE 300
    - TSE in January 1984
  - FT-SE 100
    - LIFFE in May 1984
  - BOVESPA
    - BOVESPA in February 1986

Stock index futures

- Major derivatives exchanges often have futures contracts on multiple indexes.
  - “First-mover” usually gathers lion’s share of trading volume.
    - One index is preeminent.
      - US: S&P 500
      - Europe: Euro Stoxx 50

Exchange products

- Stock index futures
  - Dominant US exchange is CME Group
    - https://www.cmegroup.com/trading/products/

Exchange products

Product name	Index	Volume	Open interest	Turnover
E-mini S&P 500 Futures	US	1,466,350	2,762,774	53.1%
E-mini Russell 2000 Index Futures	US	116,286	490,965	23.7%
S&P 500 Total Return Index Futures	US	3,000	272,701	1.1%
E-mini Nasdaq-100 Futures	US	545,487	206,243	264.5%
E-mini Dow ($5) Futures	US	238,563	104,903	227.4%
S&P 500 Annual Dividend Index Futures	US	3,786	92,798	4.1%
E-mini S&P MidCap 400 Futures	US	11,871	78,286	15.2%
Micro E-mini S&P 500 Index Futures	US	291,768	56,399	517.3%
Nikkei/Yen Futures	International	49,610	54,924	90.3%
Dow Jones Real Estate Futures	US	757	40,410	1.9%
Micro E-mini Nasdaq-100 Index Futures	US	256,361	38,119	672.5%
S&P 500 Futures	US	1,640	37,628	4.4%
E-mini Financial Select Sector Futures	Select Sector	1,267	31,965	4.0%

Highest open interest.

Exchange products

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Pit-traded contract is 5 times size.

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Variety of different stock indexes

Exchange products

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Major US stock indexes, international indexes, sector indexes

Exchange products

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Low values are buy-and-hold futures.
High values are day-trading futures.

Exchange products

- Supporting file: Index return statistics.xlsx
  - Downloaded daily data from Datastream for period 20091231 through 20191231.
    - Eliminated non-trading days.

Stock index futures

Summary statistics of daily index returns for period 20091231 through 20192131							
	S&P 500	RUSS2000	Nasdaq 100	Dow	Midcap 400	DJ RE	S&P Fin
n	2,516	2,516	2,516	2,516	2,516	2,516	2,516
Mean	0.000483	0.000504	0.000711	0.000465	0.000528	0.000365	0.000498
StDev	0.009953	0.013373	0.010964	0.009288	0.011610	0.015936	0.016699
Skewness	-0.315213	-0.135942	-0.245333	-0.298776	-0.283405	0.373483	0.446630
Minimum	-0.068958	-0.093317	-0.063053	-0.057061	-0.086123	-0.115888	-0.121282
Median	0.000362	0.000554	0.000773	0.000374	0.000606	0.000382	0.000273
Maximum	0.068366	0.080660	0.063621	0.066116	0.071101	0.152206	0.163312
CAGR	12.93%	13.54%	19.63%	12.42%	14.24%	9.64%	13.37%
Volalitily	15.80%	21.23%	17.41%	14.74%	18.43%	25.30%	26.51%
HPR	236.70%	255.43%	498.47%	221.85%	277.74%	150.62%	249.93%

Nasdaq had highest price appreciation.

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S&P Financial Sector index had highest volatility.

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Major indexes had negative skewness.

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Sector indexes had positive skewness.

Stock index futures

Correlation estimates of daily index returns during period 20091231 through 20192131							
	S&P 500	RUSS2000	Nasdaq 100	Dow	Midcap 400	DJ RE	S&P Fin
S&P 500	1						
RUSS2000	0.918	1					
Nasdaq 100	0.925	0.857	1				
Dow	0.975	0.871	0.877	1			
Midcap 400	0.950	0.969	0.877	0.906	1		
DJ RE	0.761	0.750	0.656	0.717	0.780	1	
S&P Fin	0.872	0.817	0.723	0.841	0.840	0.803	1

For major indexes, strongest correlation is between S&P and Dow.

Stock index futures

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For major indexes, strongest correlation is between S&P and Dow.
- Implies Dow futures market will not compete for hedging purposes.

Stock index futures

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For major indexes, weakest correlation is between S&P and Russell 2000.
- Implies Russell 2000 futures market can compete for hedging purposes.

Stock index futures

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Stock index futures

- Most active index futures in U.S. is E-mini S&P 500 contract.
  - Is 50 times index level.
  - Expires at open on third Friday of month.
  - Is cash-settled to special index level based on opening trade prices of each of S&P 500 stocks.
  - Weekly expirations have been launched but garner little trading volume.

Composition of stock indexes

- Value-weighted arithmetic indexes
  - S&P 500, Russell 2000, S&P 400, Nasdaq 100
- Price-weighted arithmetic index
  - Dow

Value-weighted index

- S&P 500 index:
  - Included 500 stocks for first time in 1957.
  - Initial divisor was computed using average share prices of index stocks during period 1941-3.
    - Base index level was set equal to 10.
    - Current index level is about 3326.50.
    - Price appreciation of about 33,165%.

Price-weighted index

- DJIA 30:
  - Began with 12 “blue-chip” stocks on May 26, 1896.
    - Average price on that day was 40.94.
  - Increased to 20 stocks in 1916.
  - Increased to 30 stocks in 1928.

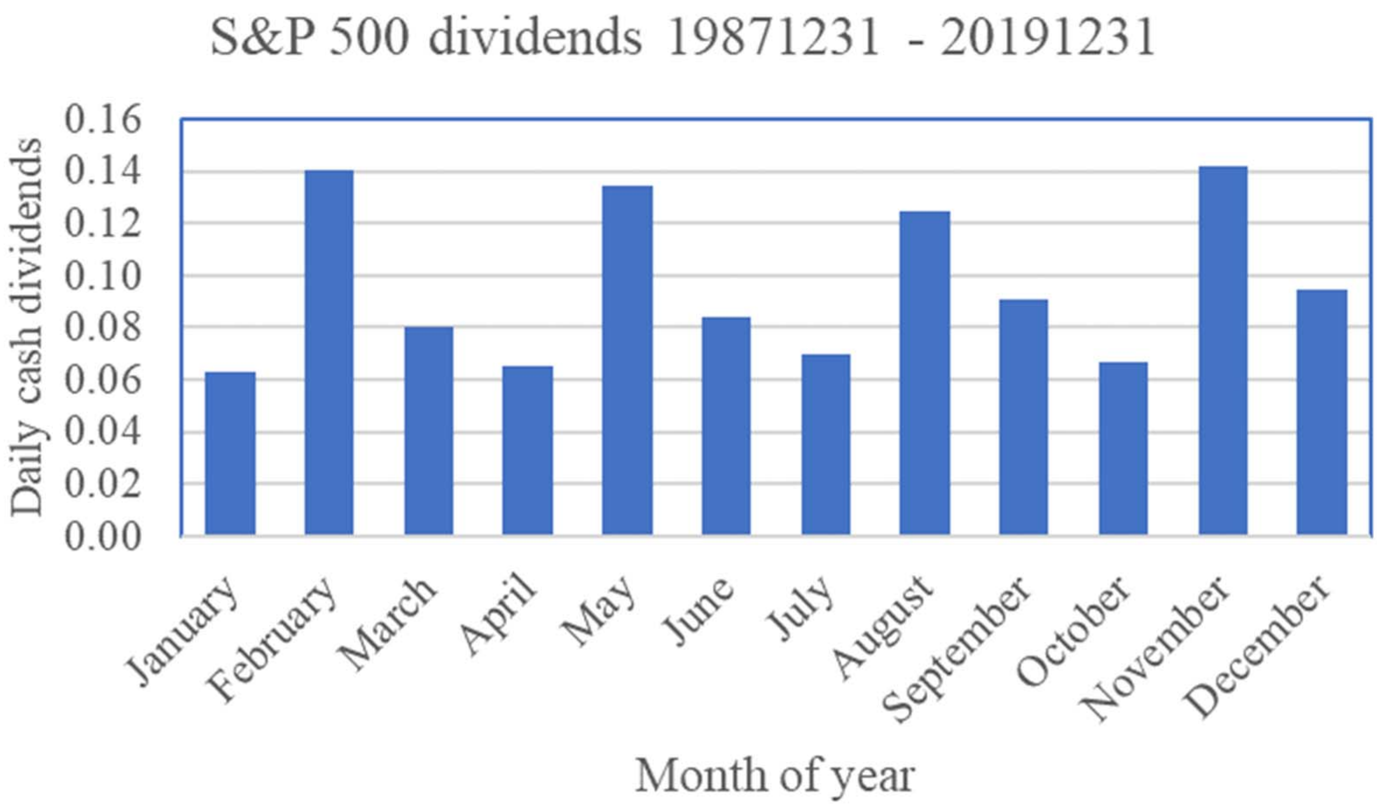
Stock index futures valuation

- Earlier, we developed two versions of cost of carry relation.
  - Income is constant continuous rate.
  - Income is discrete payments.
- Which one should be used for stock index futures?

S&P 500 dividends

- Supporting file: S&P 500 index dividends.xlsx
  - Downloaded daily data from Datastream for period 19871231 through 20191231.
    - Eliminated non-trading days.

Dividend patterns



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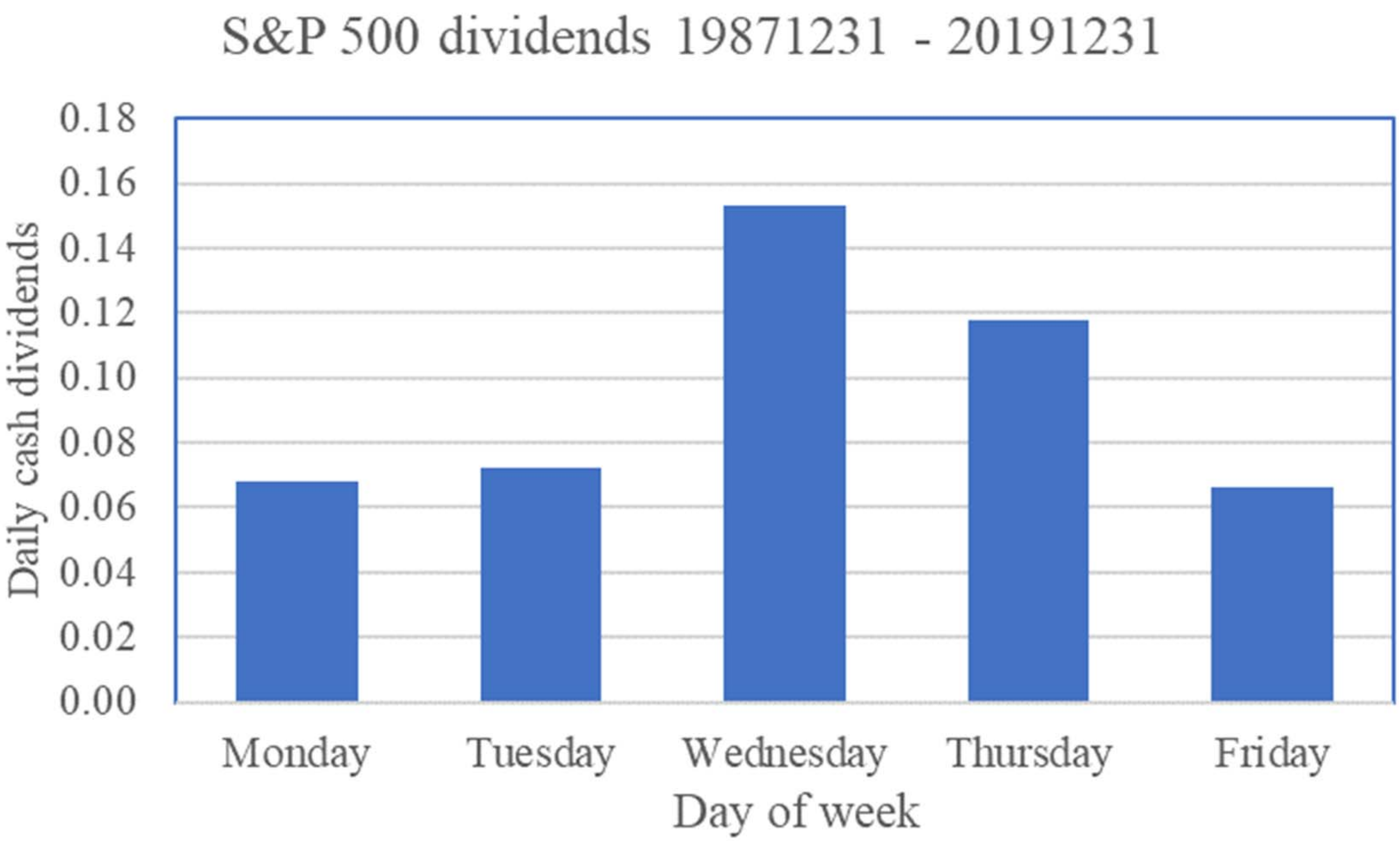

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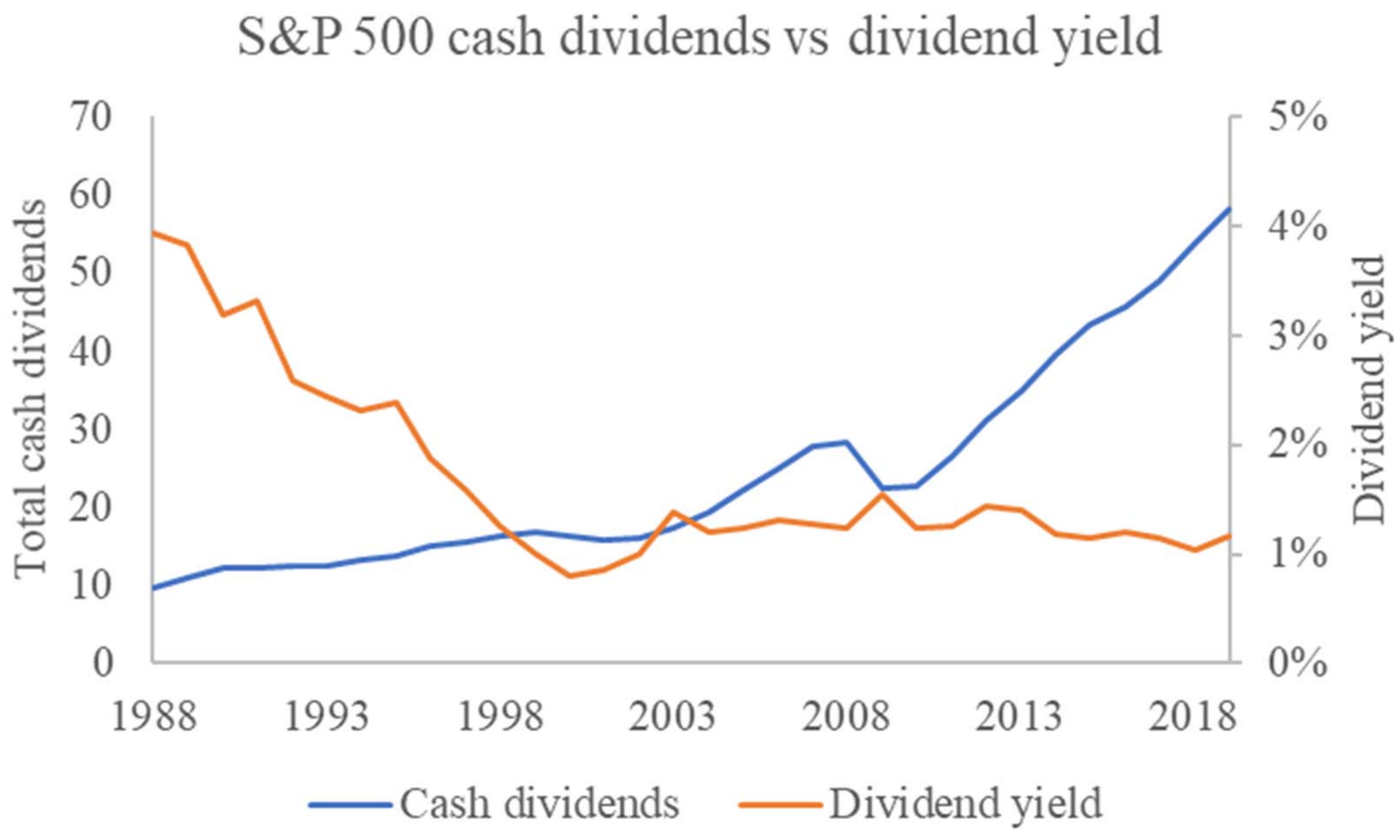

Stock index futures valuation

- Need to handled dividends as discrete flows.
- Net cost of carry relation for stock index futures is

$$F = Se^{rT} - \sum_{i=1}^{n} D_i e^{r(T - t_i)}$$

Stock index arbitrage

- Trading costs for index arbitrageurs are about 1.50 index points.
  - Will execute an arbitrage if

$$F < Se^{rT} - \sum_{i=1}^{n} D_i e^{r(T-t_i)} - 1.50$$

  - Buy futures, sell index portfolio, invest proceeds in T-bills.

Stock index arbitrage

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  - Will execute an arbitrage if

$$F > Se^{rT} - \sum_{i=1}^{n} D_i e^{r(T-t_i)} - 1.50$$

  - Sell futures, borrow, and buy index portfolio.

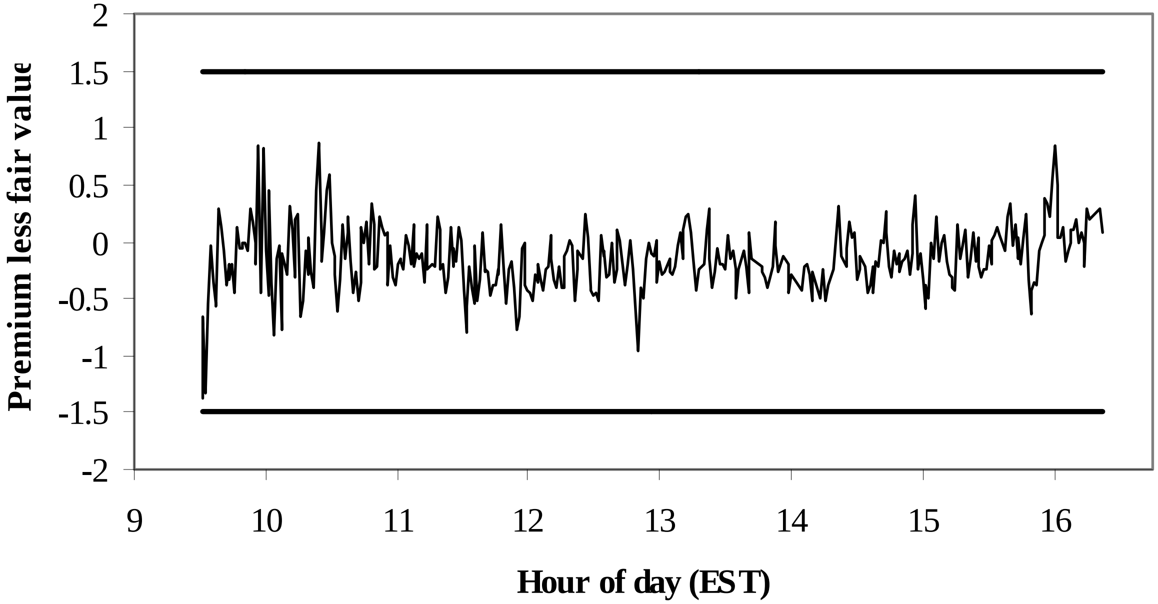
Stock index arbitrage

- How efficient is index arbitrage?
  - On a minute-by-minute basis on 8/29/03, computed

$$\text{Mispricing} = F - \left[Se^{rT} - \sum_{i=1}^{n} D_i e^{r(T-t_i)} \right]$$

Stock index arbitrage

- How efficient is index arbitrage?
  - Mis-pricing and arbitrage bounds on 8/29/03



Stock index arbitrage

- Buying or selling of index stocks must be *simultaneous*.
  - Requires use of computer-generated orders.
  - Referred to as *basket trading* or *program trading*.

Hedging with index futures

- Manage stock portfolio risk by buying and selling index futures contracts.
  - Assume objective is to minimize risk of hedged portfolio return (subject to given level of return) using available futures contracts.

Hedging with index futures

- Elements of least-risk hedging:
  - Identify least-risk hedge ratio.
  - Show equivalence of hedge ratio to OLS regression slope coefficient.
  - Discuss estimation issues.
  - Generalize model to multiple sources of risk.

Hedging with index futures

- Notation:

$V_0 =$ initial value of portfolio to be hedged

$\tilde{V}_T =$ uncertain portfolio value at time T

$\tilde{R}_V = \ln\left(\frac{\tilde{V}_T}{V_0}\right) =$ return on portfolio to be hedged

$F_0 =$ initial futures price

$\tilde{F}_T =$ uncertain futures price at time T

$\tilde{R}_F = \ln\left(\frac{\tilde{F}_T}{F_0}\right) =$ return on futures

$n_F =$ number of futures contracts bought (+) or sold (–)

$\tilde{R}_H =$ return on hedged portfolios

Hedging with index futures

- Hedged portfolio return is

$$\tilde{R}_H = \tilde{R}_V + n_F \tilde{R}_F$$

- Risk of hedged portfolio return is

$$\sigma_H^2 = \sigma_V^2 + n_F^2 \sigma_F^2 + 2 n_F \sigma_{V,F}$$

- Least-risk hedge is determined by

$$\frac{d\sigma_H^2}{n_F} = 2 n_F^* \sigma_F^2 + 2\sigma_{V,F} = 0$$

Hedging with index futures

- Least-risk hedge ratio is

$$n_F^* = -\frac{\sigma_{V,F}}{\sigma_F^2} = -\rho_{V,F}\left(\frac{\sigma_V}{\sigma_F}\right)$$

- Least-risk hedge ratio if portfolio being hedged underlies futures (e.g., S&P 500)?

$$n_F^* = -\rho_{V,F}\left(\frac{\sigma_V}{\sigma_F}\right) = -1$$

Hedging with index futures

- Illustration: Find least-risk hedge ratio for stock portfolio.
  - Supporting file: Stock portfolio hedge.xlsx
    - Contains daily values of stock portfolio and index stock futures prices.
    - Computes hedge ratio:
      - Analytically
      - Using regression
      - Using SOLVER

Hedging with index futures

- Step 1: Identify appropriate futures contract(s).
  - Since no futures are written on stock portfolio, identify closest, liquid substitute.
    - Examine correlation between stock portfolio and available index futures. Generally want contract with highest correlation.
    - Depth and liquidity of index futures is also important.
      - Depth ensures small price impact.
      - Liquidity (i.e., small bid/ask spread) ensures fast and cost-efficient trading.

Hedging with index futures

- Step 2: Collect historical time series.
  - Daily stock portfolio values
  - Index futures prices
    - Index futures price contract denomination is 50.

Hedging with index futures

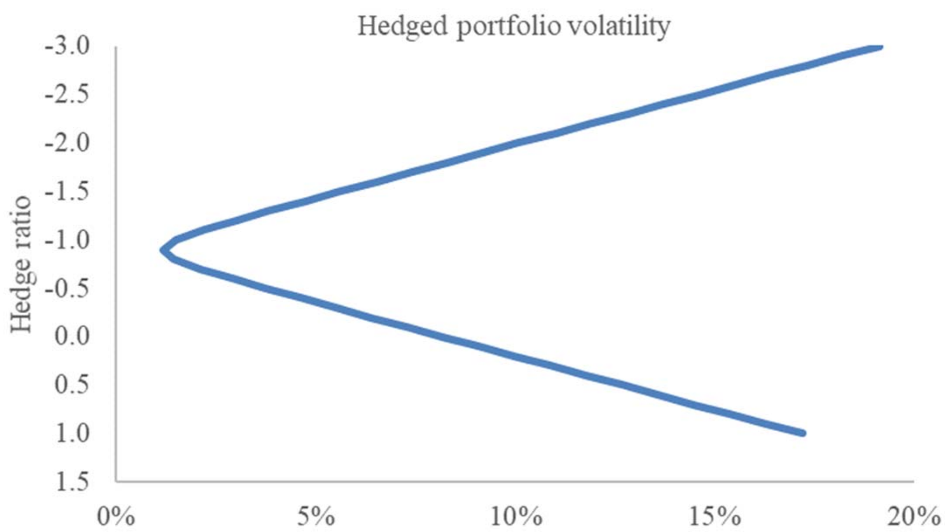
- Step 3: Estimate parameters of minimum risk hedge.
  - Compute:
    - standard deviation of stock portfolio returns
    - standard deviation of futures returns
    - correlation between stock portfolio and futures returns

Summary statistics for returns		
	Stock portfolio return	Index futures return
n	294	294
Mean	0.000405	0.000333
StDev	0.005151	0.005706
Skewness	0.009638	0.075221
Correlation	0.989811	

Hedging with index futures

- Step 4: Compute least-risk hedge ratio.

$$n_F^* = -.989811\left(\frac{.005151}{.005711}\right) = -.893543$$



Hedging with index futures

- Step 5: Determine least-risk number of futures.
  - Hedge ratio is number of futures to sell per unit of stock portfolio.
    - Need to adjust by dollar value of portfolio and dollar value of stock index futures.

$$n_F^* = -.89354\left(\frac{32,671,455}{1,152.50 \times 50}\right)$$

$$= -.89354\left(\frac{32,671,455}{1,152.50 \times 50}\right)$$

$$= -506.61 \approx -507$$

Regression approach

- OLS regression provides alternative means for risk measurement.
- Consider regression of portfolio return on futures return.

$$\tilde{R}_V = \alpha_0 + \alpha_1 \tilde{R}_F + \tilde{\varepsilon}$$

- Hedge portfolio return may be written

$$\begin{aligned}\tilde{R}_H &= \tilde{R}_V + n_F \tilde{R}_F \\ &= \alpha_0 + \alpha_1 \tilde{R}_F + \tilde{\varepsilon} + n_F \tilde{R}_F \\ &= \alpha_0 + \left(\alpha_1 + n_F\right) \tilde{R}_F + \tilde{\varepsilon}\end{aligned}$$

Regression approach

- Variance of hedged portfolio return is

$$
\begin{aligned}
Var\left(\tilde{R}_H\right) &= Var\left[\alpha_0 + \left(\alpha_1 + n_F\right)\tilde{R}_F + \tilde{\varepsilon}\right] \\
&= Var\left[\left(\alpha_1 + n_F\right)\tilde{R}_F\right] + Var\left(\tilde{\varepsilon}\right) + 2Cov\left(\left(\alpha_1 + n_F\right)\tilde{R}_F, \tilde{\varepsilon}\right) \\
&= Var\left[\left(\alpha_1 + n_F\right)\tilde{R}_F\right] + Var\left(\tilde{\varepsilon}\right)
\end{aligned}
$$

- Where is hedged portfolio risk minimized?

$$n_F^* = -\alpha_1$$

Regression approach

- Illustration: Find risk-minimizing hedge for portfolio of stocks.
  - Supporting file: Stock portfolio hedge.xlsx

Same risk-minimizing hedge ratio.

Regression Statistics	
Multiple R	0.9898
R Square	0.9797
Adjusted R Square	0.9797
Standard Error	0.0007
Observations	294

	Coeff	*StErr*	*t Stat*
Intercept	0.00011	0.00004	2.50
Futures return	0.89354	0.00752	118.79

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Adjusted R Squared is measure of *hedging effectiveness*.

	Coeff	*StErr*	*t Stat*
Intercept	0.00011	0.00004	2.50
Futures return	0.89354	0.00752	118.79

Percent of portfolio return variance explained by futures returns.

Estimation issues

- Return interval must be selected (e.g., daily, weekly, monthly, etc. returns)
  - higher frequency implies more information (good) but also more noise (bad)
  - prices for cash and futures must be simultaneous

Estimation issues

- Illustration: Suppose we hold S&P 500 index portfolio and want to hedge market risk using S&P 500 futures.
  - Already know least-risk hedge ratio is –1.

Estimation issues

- Suppose we use historical data to estimate hedge.
- S&P 500 futures data during 1997
  - 1997 had 254 trading days, which creates
    - 253 daily returns
    - 52 weekly returns
    - 26 biweekly returns
  - For weekly and biweekly returns, Wednesday closing prices are used.

Estimation issues

- Use returns of nearby futures contract.
  - When switching contract months, care must be taken to splice price change series correctly.
- Supporting file: S&P 500 hedge.xlsx

Estimation issues

- Daily, weekly, and bi-weekly regressions using S&P 500 futures data during 1997.

Hedge regressions using different return intervals

	α_0	t(α_0)	α_1	t(α_1)	Adjusted R-squared
Daily	0.0003	1.65	0.8764	70.07	0.9514
Weekly	0.0009	2.29	0.9558	55.64	0.9841
Bi-weekly	0.0016	2.61	0.9884	43.40	0.9874

Estimation issues

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Measurement error can be large.
Correct answer is 1.

Estimation issues

- CAVEAT: Parameters are estimated using past data, but we are interested in future.
  - Must have reason to believe relation is stationary.

Hedging multiple risks

- OLS approach can be generalized to handle multiple sources of risk.

$$\tilde{R}_V = \alpha_0 + \alpha_1 \tilde{R}_{F,1} + \alpha_2 \tilde{R}_{F,2} + \ldots + \alpha_n \tilde{R}_{F,n} + \tilde{\varepsilon}$$

- Set hedge ratios for all or just selected risks.

Hedging multiple risks

- Illustration: Hedge oil price risk of fund that invests primarily in oil stocks.

Hedging multiple risks

- Step 1: Identify portfolio risk exposures and find futures to proxy for each.
  - SP: S&P 500 futures (stock market risk)
  - CL: crude oil futures (petroleum price risk)
  - HO: heating oil futures (processed petroleum price risk)
  - HU: unleaded gas futures (processed petroleum price risk)

Hedging multiple risks

- Step 2: Collect historical return data.
  - Supporting file: Oil hedge.xlsx

Day	Portfolio value ($ millions)	Portfolio return	Futures prices SP	Futures prices CL	Futures prices HO	Futures prices HU	Futures returns SP	Futures returns CL	Futures returns HO	Futures returns HU
1	769.26		1100.00	40.00	0.6900	0.7500				
2	776.57	0.00946	1099.00	40.39	0.6969	0.7616	-0.00091	0.00970	0.00995	0.01535
3	782.45	0.00754	1100.55	40.50	0.7051	0.7709	0.00141	0.00272	0.01170	0.01214
4	791.62	0.01165	1101.10	41.17	0.7236	0.7878	0.00050	0.01641	0.02590	0.02169
5	791.44	-0.00023	1100.90	41.25	0.7213	0.7865	-0.00018	0.00194	-0.00318	-0.00165
6	789.32	-0.00268	1104.30	41.15	0.7265	0.7882	0.00308	-0.00243	0.00718	0.00216
7	793.38	0.00513	1109.05	40.50	0.7124	0.7670	0.00429	-0.01592	-0.01960	-0.02727
8	789.23	-0.00524	1108.40	40.68	0.7290	0.7764	-0.00059	0.00443	0.02303	0.01218
9	786.04	-0.00405	1107.25	40.16	0.7255	0.7673	-0.00104	-0.01287	-0.00481	-0.01179
10	784.78	-0.00160	1106.65	40.34	0.7339	0.7690	-0.00054	0.00447	0.01151	0.00221

Hedging multiple risks

- Step 3: Get to know properties of data.

SP volatility low during sample period.

Summary statistics				
	Futures returns			
	SP	CL	HO	HU
n	252	252	252	252
Mean	-0.000029	0.000483	0.000773	0.000609
StDev	0.002798	0.006899	0.011017	0.011879
Skewness	-0.473383	-0.162212	-0.207878	-0.265877
CAGR	-0.73%	12.95%	21.49%	16.60%
Volatility	4.44%	10.95%	17.49%	18.86%

Correlations				
	SP	CL	HO	HU
SP	1			
CL	-0.191	1		
HO	-0.135	0.773	1	
HU	-0.079	0.718	0.662	1

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Hedging multiple risks

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SP volatility low during sample period.

CL and processed products are more normal.

CL less volatility than HO or HU.

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Hedging multiple risks

- Step 3: Get to know properties of data.

What does this say about oil oil stock exposure?

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Hedging multiple risks

- Step 3: Get to know properties of data.

What does this say about oil oil stock exposure?

Are stocks:
1) Oil exploration
2) Oil refining, or
3) Oil production distribution?

Summary statistics

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Hedging multiple risks

- Step 4: Run OLS regression.

$$\tilde{R}_V = \alpha_0 + \alpha_1 \tilde{R}_{F,1} + \alpha_2 \tilde{R}_{F,2} + \ldots + \alpha_n \tilde{R}_{F,n} + \tilde{\varepsilon}$$

Regression Statistics	
Multiple R	0.5826
R Square	0.3394
Adjusted R Square	0.3287
Standard Error	0.0060
Observations	252

	Coeff	*StErr*	*t Stat*
Intercept	0.00002	0.00038	0.05
SP	1.55502	0.13904	11.18
CL	0.21055	0.09773	2.15
HO	-0.03085	0.05610	-0.55
HU	-0.00062	0.04764	-0.01

Hedging multiple risks

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Hedge is not (will not be) particularly effective.

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HO and HU returns have little effect on portfolio return.

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HO and HU returns have little effect on portfolio return.

Drop HO and HU and re-run regression.

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HU	-0.00062	0.04764	-0.01

Hedging multiple risks

- Step 5: Drop heating oil and crude oil and re-run regression.

Adjusted R-squared is increased.

Regression Statistics	
Multiple R	0.5818
R Square	0.3385
Adjusted R Square	0.3332
Standard Error	0.0060
Observations	252

	Coeff	*StErr*	*t Stat*
Intercept	0.00001	0.00038	0.04
SP	1.55328	0.13808	11.25
CL	0.17158	0.05600	3.06

Hedging multiple risks

- Step 5: Drop heating oil and crude oil and re-run regression.

Adjusted R-squared is increased.

Both SP and CL returns explain portfolio returns.

Regression Statistics	
Multiple R	0.5818
R Square	0.3385
Adjusted R Square	0.3332
Standard Error	0.0060
Observations	252

	Coeff	*StErr*	*t Stat*
Intercept	0.00001	0.00038	0.04
SP	1.55328	0.13808	11.25
CL	0.17158	0.05600	3.06

Hedging multiple risks

- Step 6: Find number of crude oil futures to sell.

$$n_F^* = -.17158\left(\frac{779,520,000}{45.18 \times 1,000}\right) = -2,960.36 \approx -2,960$$

Estimation issues

- Missing variable bias: Regress on crude oil futures return only.

Regression Statistics	
Multiple R	0.0482
R Square	0.0023
Adjusted R Square	-0.0017
Standard Error	0.0074
Observations	252

	Coeff	*StErr*	*t Stat*
Intercept	0.00003	0.00047	0.06
CL	0.05136	0.06738	0.76

$$n_F^* = -.05136\left(\frac{779{,}520{,}000}{45.18\times 1{,}000}\right) = -886.22 \approx -886$$

Estimation issues

- Missing variable bias: Recall return correlations.

Correlations				
	SP	CL	HO	HU
SP	1			
CL	-0.191	1		
HO	-0.135	0.773	1	
HU	-0.079	0.718	0.662	1

Absence of SP means that CL, to some degree, also picks up effect of CL because SP and CL are correlated.

Negative correlation implies downward bias.

Lesson summary

- Stock index futures were introduced in U.S. in 1982; stock index options in 1983.
- Construction of stock indexes
  - value-weighted indexes
  - price-weighted indexes
- Cash dividends
  - use discrete cash flows in valuation

Lesson summary

- Index arbitrage
  - Program trading
- *OLS regression* of portfolio returns on futures returns provides estimate of minimum-risk hedge ratio.
- *Multiple regression* can be used to estimate hedge ratios for portfolios with multiple sources of risk.

Lesson summary

- Steps in setting multiple risk portfolio hedge:
  - Identify portfolio risk exposures and find futures to proxy for each.
  - Collect historical return series.
  - Estimate OLS regression.
    - Use coefficient estimates to hedge selected exposures.

Lesson summary

- Careful data analysis is necessary to properly estimate hedge ratios. Examined effects of:
  - Data frequency
  - Missing variables bias
    - Failing to include relevant explanatory variable is much more serious than including irrelevant explanatory variable.