Compensation contracts

Compensation contracts

- Purpose:
  - Describe and value compensation contracts based on firm’s share price.
  - Included are:
    - Employee stock options (ESOs)
      - Standard (fixed exercise price)
      - Indexed
    - Employee stock purchase plans (ESPPs)

Standard ESOs

- Standard employee stock options (ESOs) are call options issued by firm.
  - Are at-the-money at time of issuance.
  - Have times to expiration of ten years.
  - Have *vesting period* when issued.
    - Cannot be exercised during first three years.
      - If employee leaves firm during vesting period, options are forfeit.
      - After vesting date, options can be exercised at any time.
  - Are *non-transferable*.
    - Only way to monetize is to exercise.

Standard ESOs

- Exercise of ESOs dilutes value of existing shares.
  - Like warrants and convertible bonds.
  - Technically, applying BSM model is incorrect.
  - For most ESOs, existing shareholder base is so large, so dilution factor is small.
- Accepted practice to apply BSM call option valuation equations/methods.
  - Need estimates of interest rate, expected dividend stream, and expected volatility rate.

Standard ESOs

- Illustration. Value standard ESO.
  - Firm:
    - share price, 50
    - dividend yield rate, 1%
    - volatility rate, 50%
  - Option:
    - exercise price, 50
    - years to expiration, 10

Standard ESOs

- Illustration. Value standard ESO.
  - Compute value of:
    - European-style call using BSM model
    - European-style call using binomial method
    - American-style call using binomial method

Standard ESOs

- Illustration. Value standard ESO.

B13 f_x =OV_OPTION_VALUE(B3,B11,B12,B8,B4,B5,"C","E")

	A	B	C	D	E
1	**Standard employee stock option**				
2	***Stock***				
3	Stock price (S_1)	50			
4	Income rate (i_1)	1.00%			
5	Volatility rate (σ_1)	50.00%			
6					
7	***Market***				
8	Interest rate (r)	4.00%			
9					
10	***Standard ESO (pay X)***				
11	Exercise price (X)	50			
12	Years to expiration (T)	10			
13	European-style ESO value (analytical)	28.664			
14	No. of time steps	100			
15	Method (1=CRR,2=JR,3=simplified CRR)	1			
16	European-style ESO value (binomial)	28.541			
17	Approximation error	-0.123			
18	American-style ESO value (binomial)	29.047			
19	Early exercise premium	0.506			

Standard ESOs

- Illustration. Value standard ESO.

B18 =OV_APPROX_STD_OPT_BIN(B3,B11,B12,B8,B4,B5,B14,"C","A",B15)

	A	B	C	D	E	F	G
1	**Standard employee stock option**						
2	***Stock***						
3	Stock price (S_1)	50					
4	Income rate (i_1)	1.00%					
5	Volatility rate (σ_1)	50.00%					
6							
7	***Market***						
8	Interest rate (r)	4.00%					
9							
10	***Standard ESO (pay X)***						
11	Exercise price (X)	50					
12	Years to expiration (T)	10					
13	European-style ESO value (analytical)	28.664					
14	No. of time steps	100					
15	Method (1=CRR,2=JR,3=simplified CRR)	1					
16	European-style ESO value (binomial)	28.541					
17	Approximation error	-0.123					
18	American-style ESO value (binomial)	29.047					
19	Early exercise premium	0.506					

Standard ESOs

- For many ESO holders, value of ESOs represent significant portion of their wealth.
  - Exercising early offers opportunity to cash-in and diversify relatively undiversified portfolio.
  - Empirical suggests employees tend to exercise ESOs when stock price reaches certain multiples of option’s exercise price.
  - This type of behavior can be accommodated easily with lattice-based valuation procedures like binomial method.

Standard ESOs

- Illustration. Value standard ESO.
  - Using binomial method, compute value of:
    - American-style call
    - American-style call with automatic exercise when stock price reaches two times exercise price

Standard ESOs

- Illustration. Value standard ESO.

B17 =OV_APPROX_STD_OPT_BIN_BND(B3,B16,B11,B12,B8,B4,B5,B13,"C","A",B14)

	A	B	C	D	E	F
1	**Standard employee stock option with upper bound**					
2	***Stock***					
3	Stock price (S_1)	50				
4	Income rate (i_1)	1.00%				
5	Volatility rate (σ_1)	50.00%				
6						
7	***Market***					
8	Interest rate (r)	4.00%				
9						
10	***Standard ESO (pay X)***					
11	Exercise price (X)	50				
12	Years to expiration (T)	10				
13	No. of time steps	100				
14	Method (1=CRR,2=JR,3=simplified CRR)	1				
15	American-style ESO value	29.047				
16	Upper bound (2 times exercise price)	100				
17	American-style ESO value with upper bound	23.373				
18	Value difference	5.674				

Standard ESOs

- Illustration. Value standard ESO.
  - Supporting file: Compensation contracts.xls

Indexed ESOs

- Standard ESOs are issued at the money and have fixed exercise price.
  - Peculiar reward structure.
    - Manager can benefit from bull market even though firm performed poorly relative to competitors.

Indexed ESOs

- Simple alternative is to create indexed option.
  - Manager is awarded only insofar as she outperforms index.
  - Index can be:
    - Portfolio of stocks in same industry.
    - Firm’s chief competitor.

Indexed ESOs

- Can modify BSM model to account for uncertain exercise price, i.e., index level.
  - Option holder receives in cash difference between firm's share price and index level.

$$\tilde{c}_T = \begin{cases} \tilde{S}_{1,T} - \tilde{S}_{2,T} & \text{if } \ S_{1,T} > S_{2,T} \\ 0 & \text{if } \ S_{1,T} \le S_{2,T} \end{cases}.$$

Indexed ESOs

- Cannot apply BSM mechanics directly.
  - Problem: Both asset prices are log-normally distributed, so difference cannot be log-normally distributed.

Indexed ESOs

- Restructure payoffs to have fixed exercise price.

$$\tilde{c}_T = \begin{cases} \tilde{S}_{1,T} - \tilde{S}_{2,T} & \text{if} \quad S_{1,T} > S_{2,T} \\ 0 & \text{if} \quad S_{1,T} \leq S_{2,T} \end{cases}.$$

$$\tilde{c}_T / \tilde{S}_{2,T} = \begin{cases} \tilde{S}_{1,T} / \tilde{S}_{2,T} - 1 & \text{if} \quad S_{1,T} / S_{2,T} > 1 \\ 0 & \text{if} \quad S_{1,T} / S_{2,T} \leq 1 \end{cases}.$$

Indexed ESOs

- Stock price is log-normally distributed.
  - What is distribution of ratio of stock prices?

$$\tilde{S}_{1,T} / \tilde{S}_{2,T}$$

Indexed ESOs

- Stock price is log-normally distributed.
  - What is distribution of ratio of stock prices?

$$\tilde{S}_{1,T} / \tilde{S}_{2,T}$$

  - Answer: log-normal

Indexed ESOs

- Stock price is log-normally distributed.
  - What is distribution of log of ratio of stock prices?

$$\ln\left(\tilde{S}_{1,T} / \tilde{S}_{2,T}\right) = \ln \tilde{S}_{1,T} - \ln \tilde{S}_{2,T}$$

  - <u>Answer:</u> Normal

Indexed ESOs

- Stock price is log-normally distributed.
  - What is volatility rate of log of ratio of stock prices?

$$\begin{aligned} Var\left[\ln\left(\tilde{S}_{1,T} / \tilde{S}_{2,T}\right)\right] &= Var\left(\ln \tilde{S}_{1,T} - \ln \tilde{S}_{2,T}\right) \\ &= Var\left(\ln \tilde{S}_{1,T}\right) + Var\left(\ln \tilde{S}_{2,T}\right) - 2Cov\left(\ln \tilde{S}_{1,T}, \ln \tilde{S}_{2,T}\right) \\ &= \sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2 \end{aligned}$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2}$$

Indexed ESOs

- Value of indexed option

$$c = S_1 e^{-i_1 T} N\left(d_1\right) - S_2 e^{-i_2 T} N\left(d_2\right)$$

$$d_1 = \frac{\ln\left(S_1 e^{-i_1 T} / S_2 e^{-i_2 T}\right) + .5\sigma^2 T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2}$$

Indexed ESOs

- Illustration. Firm is considering two types of stock option designs for its employees.
  - Option with fixed exercise price.
  - Option with indexed exercise price.

Indexed ESOs

- Illustration. Evaluate competing option awards.
  - Firm attributes:
    - share price, 50
    - dividend yield rate, 1%
    - volatility rate, 50%.

Indexed ESOs

- <u>Illustration</u>. Evaluate competing option awards.
  - <u>Alternative 1:</u> Fixed exercise price ESO.
    - exercise price, 50
    - time to expiration, 10 years.
  - <u>Alternative 2:</u> Indexed exercise price ESO.
    - index level, 50
    - dividend yield rate, 1%
    - volatility rate, 40%.
    - correlation between stock and index returns is 80%.

Indexed ESOs

- Illustration. Compute values.

E14 =OV_NS_EXCHANGE_OPTION(B3,E3,E13,B4,E4,B5,E5,B6,"C")

	A	B	C	D	E	F
1	**Indexed employee stock option valuation**					
2	***Stock***			***Index***		
3	Stock price (S_1)	50		Index level (S_2)	50	
4	Income rate (i_1)	1.00%		Income rate (i_2)	1.00%	
5	Volatility rate (σ_1)	50.00%		Volatility rate (σ_2)	40.00%	
6	Correlation between returns (ρ)	0.80				
7						
8	***Market***					
9	Interest rate (r)	4.00%				
10						
11	***Standard ESO (pay X)***			***Indexed ESO (pay S_2)***		
12	Exercise price (X)	50		Exercise price (S_2)	50	
13	Years to expiration (T)	10		Years to expiration (T)	10	
14	Value	28.664		Value	16.502	

Indexed ESOs

- Illustration. Since values are different can afford to give more Indexed ESOs. Suppose firm is trying to decide whether to award 5,000 fixed exercise price calls or 8,000 indexed calls. What is cost?

Standard ESO (pay X)		***Indexed ESO (pay S_2)***	
Exercise price (*X*)	50	Exercise price (S_2)	50
Years to expiration (*T*)	10	Years to expiration (*T*)	10
Value	28.664	Value	16.502
Number of options awarded	5,000	Number of options awarded	8,000
Cost of award	143,319		132,013

Indexed ESOs

- Illustration. Objective of ESO is to award employees for out-performance.
  - After one year, assume share price is 55 and index is 60.
    - share price has increased by 10%, however, index has increased by 20%.
    - poor performance

Indexed ESOs

- Illustration. Objective of ESO is to award employees for out-performance.

Standard ESO (pay X)		***Indexed ESO (pay S_2)***	
Exercise price (X)	50	Exercise price (S_2)	50
Years to expiration (T)	10	Years to expiration (T)	10
Value	28.664	Value	16.502
Number of options awarded	5,000	Number of options awarded	8,000
Cost of award	143,319		132,013
Years elapsed	1	Years elapsed	1
Stock price (S_1)	55	Index level (S_2)	60
Years to expiration (T)	9	Years to expiration (T)	9
Value	31.529	Value	16.045
Percent change	9.99%	Percent change	-2.77%

Indexed ESOs

- Illustration. Objective of ESO is to award employees for out-performance.
  - After one year, assume share price is 48 and index is 40.
    - share price has fallen by 4%, however, index has fallen by 20%.
    - excellent performance

Indexed ESOs

- Illustration. Objective of ESO is to award employees for out-performance.

Standard ESO (pay X)		**Indexed ESO (pay S_2)**	
Exercise price (X)	50	Exercise price (S_2)	50
Years to expiration (T)	10	Years to expiration (T)	10
Value	28.664	Value	16.502
Number of options awarded	5,000	Number of options awarded	8,000
Cost of award	143,319		132,013
Years elapsed	1	Years elapsed	1
Stock price (S_1)	48	Index level (S_2)	40
Years to expiration (T)	9	Years to expiration (T)	9
Value	26.227	Value	17.887
Percent change	-8.50%	Percent change	8.40%

ESPPs

- Employee stock purchase plans (ESPPs)
  - Allow holder to buy stock at discount within certain period of time.
  - Discount is usually 15%
  - Term of investment period is typically six months
  - Allow holder to apply discount to either:
    - End-of-period stock price or
    - Beginning-of-period price, whichever is less.
    - Called *lookback option*.

ESPPs

- To value ESPP, write terminal value.
  - Assume k is discount as proportion of stock price and investment period ends at time T.

$$ESPP_T = \begin{cases} \tilde{S}_T - (1-k)S & \text{if } S_T > S \\ \tilde{S}_T - (1-k)\tilde{S}_T & \text{if } S_T \le S \end{cases}$$

$$= \begin{cases} k\tilde{S}_T + (1-k)\left(\tilde{S}_T - S\right) & \text{if } S_T > S \\ k\tilde{S}_T & \text{if } S_T \le S \end{cases}$$

ESPPs

- Apply valuation-by-replication.
  - Buy k stocks
  - Buy $(1-k)$ call options with exercise price of S and time until expiration of T.

$$\text{Portfolio}_T = \begin{cases} k\tilde{S}_T + (1-k)\left(\tilde{S}_T - S\right) & \text{if } S_T > S \\ k\tilde{S}_T + 0 & \text{if } S_T \le S \end{cases}$$

ESPPs

- Apply valuation-by-replication.
  - Sum known values.

$$ESPP = kS + (1-k)\left[SN(d_1) - Se^{-rT}N(d_2)\right]$$

ESPPs

- Illustration:
  - Value ESPP that allows you to buy 10,000 shares of firm's stock at 15% discount at today's price or at market price in six months.
    - current stock price, 50
    - volatility rate, 40%
    - risk-free interest rate, 5%

ESPPs

- Illustration:
  - Substitute parameters.

$$ESPP = .15(50) + (1 - .15)\left[50N(d_1) - 50e^{-.05(.5)}N(d_2)\right] = 12.764$$

$$d_1 = \frac{\ln\left(50e^{.05(.5)}/50\right) + .5\left(.40^2\right).5}{.40\sqrt{.5}}$$

$$d_2 = d_1 - .40\sqrt{.5}$$

ESPPs

- Illustration:
  - Use OPTVAL function.

    OV_STOCK_OPTION_ESPP(*s*, *k*, *t*, *r*, *v*),

    where *s* is stock price, *k* is discount, *t* is length of investment period, *r* is risk-free interest rate, and *v* is volatility rate.

ESPPs

- Illustration:

B12 | =OV_STOCK_OPTION_ESPP(B3,B10,B11,B7,B4)

	A	B	C	D
1	**Valuation of ESPP with lookback provision**			
2	***Stock***			
3	Price (*S*)	50		
4	Volatility rate (σ)	40.00%		
5				
6	***Market***			
7	Interest rate (*r*)	5.00%		
8				
9	***ESPP***			
10	Percent discount (*k*)	15.00%		
11	Years to expiration (*T*)	0.50		
12	Value	12.764		
13				
14	***Intermediate computations***			
15	(a) stock price S	50.000		
16	(b) borrow (1-k)Sexp(-rT)	-41.451		
17	(c) buy (1-k) puts	4.214		
18	Total	12.764		

Lesson summary

- Compensation contracts can be significant liability, and need to be valued.
- Reasonable valuations can be performed using BSM model/methods.
- Show how to value:
  - Standard ESOs
  - Indexed ESOs (becoming increasingly popular)
  - ESPPs