


AIM 04 Performance measurement

AIM 04.2 Single-factor risk-adjusted performance

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Risk-adjusted performance

- Context:
 - Risk-adjusted performance evaluation is critical in investment management.
 - Are passive ETP managers matching risk and performance of benchmark index?
 - Positive tracking error may result from additional risk.
 - Negative tracking error may result from less risk.
 - Can active portfolio managers outperform benchmarks after accounting for risk and management fees?

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Risk-adjusted performance

- Purpose:
 - Describe and discuss four traditional portfolio performance measures. All are based on Sharpe-Lintner capital asset pricing model (CAPM).
 - Two are based on CML.
 - Sharpe ratio
 - M-squared
 - Two are based on SML.
 - Treynor ratio
 - Jensen's alpha

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Risk-adjusted performance

- Purpose:
 - Show two other measures that are ad hoc, albeit intuitive, and are widely accepted in practice.
 - One is based on excess return regression.
 - Treynor/Black appraisal ratio
 - One is based on ad hoc measure of risk.
 - Sortino ratio
 - Introduce concept called portable alpha.
 - Direct by-product of Jensen's alpha.

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CAPM assumptions

- Markowitz (*JF* 1952) assumed n risky assets.
- Tobin (*REST* 1958) showed link between diminishing positive marginal utility of wealth and risk averse indifference curves.
- Sharpe (*JF* 1964) and Lintner (*REST* 1965) added risk-free cash equivalents to set of securities in capital market.
 - Assumed individuals can borrow or lend at risk-free interest rate, r .

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CAPM results/predictions

- CAPM has three main results.

- 1. Capital market line (*)

$$E_P = r + (E_M - r) \left(\frac{\sigma_P}{\sigma_M} \right)$$

- 2. Composition of market portfolio

$$X_i = V_i / V_M$$

- 3. Security market line (*)

$$E_i = r + (E_M - r) \beta_i$$

(*) denotes two CAPM results that are foundation of traditional performance measures.

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Total risk measures

- Two measures are based on total risk (CML).

Expectations

Realizations

Sharpe ratio

$$E(SR) \equiv \frac{E_S - r}{\sigma_S} = \frac{E_M - r}{\sigma_M}$$

$$SR \equiv \frac{\bar{R}_S - \bar{R}_F}{\hat{\sigma}_S} = \frac{\bar{R}_M - \bar{R}_F}{\hat{\sigma}_M}$$

M-squared

$$E(M^2) \equiv E_S - r - \left(\frac{E_M - r}{\sigma_M} \right) \sigma_S = 0$$

$$M^2 \equiv \bar{R}_S - \bar{R}_F - \left(\frac{\bar{R}_M - \bar{R}_F}{\hat{\sigma}_M} \right) \hat{\sigma}_S$$

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Systematic risk measures

- Two measures are based on beta (SML).

Expectations

Realizations

Treynor ratio

$$E(TR) \equiv \frac{E_S - r}{\beta_S} = E_M - r$$

$$TR \equiv \frac{\bar{R}_S - \bar{R}_F}{\hat{\beta}_S} = \bar{R}_M - \bar{R}_F$$

Jensen's alpha

$$E(\alpha) \equiv E_S - r - (E_M - r)\beta_S = 0$$

$$\hat{\alpha} \equiv \bar{R}_S - \bar{R}_F - (\bar{R}_M - \bar{R}_F)\hat{\beta}_S$$

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Appraisal ratio

- Measures risk-adjusted reward from active management.
 - Jensen's alpha divided by residual risk (i.e., standard error of regression estimate).

$$AR = \frac{\hat{\alpha}}{\hat{\sigma}_\varepsilon}$$

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Sortino ratio

- Measures Sharpe ratio using down-side risk.
 - CAPM assumes individuals are equally unhappy about large positive returns as large negative returns.
 - Some argue that only latter matters.
 - Define risk as:

$$\sigma_{S,semi-deviation} = \sqrt{\frac{\sum_{t=1}^T \min(R_{S,t} - R_{F,t}, 0)^2}{T}}$$

$$\text{Sortino ratio} = \frac{\bar{R}_S - \bar{R}_F}{\hat{\sigma}_{S,semi-deviation}}$$

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Illustration

- Asked to evaluate risk-adjusted performance of iShares US Technology ETF (IYW).

- Support file: IYW fact sheet.pdf



iShares U.S. Technology ETF
Fact Sheet as of 12/31/2022

KEY FACTS

Fund Launch Date	05/15/2000
Expense Ratio	0.39%
Benchmark	Russell 1000 Technology RIC 22.5/45 Capped Index

30 Day SEC Yield	0.56%
Number of Holdings	140
Net Assets	\$7,827,660,617

Ticker	IYW
CUSIP	464287721
Exchange	NYSE Arca

TOP HOLDINGS (%)

APPLE INC	17.79
MICROSOFT CORP	16.51
ALPHABET INC CLASS A	5.13
ALPHABET INC CLASS C	4.58
NVIDIA CORP	4.22
META PLATFORMS INC CLASS A	3.65
BROADCOM INC	3.22
ADOBE INC	2.36
TEXAS INSTRUMENT INC	2.26
SALESFORCE INC	1.93
	61.65

Holdings are subject to change.

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Data

- Downloaded 3 years of daily total return index data from Bloomberg.

- Eliminated non-US trading days.
 - Support file: IYW analysis.xlsx
 - IYW: iShares Technology ETF
 - VOO: Vanguard S&P 500 ETF
 - SPX: S&P 500 index
 - VTI: Vanguard Total Stock Market ETF
 - CRSPTMT: CRSP Total market index

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Analysis

- Compute summary statistics.

Return summary statistics						
Description	IYW	VOO	SPX	VII	CRSPTMT	EFFR
<i>n</i>	756	756	756	756	756	756
Mean (daily)	0.00035	0.00029	0.00029	0.00027	0.00027	0.00003
StDev (daily)	0.02080	0.01608	0.01611	0.01627	0.01645	0.00006
Skewness	-0.43874	-0.73061	-0.74355	-0.74483	-0.83969	3.44605
Autocorrelation	-0.18844	-0.20893	-0.21618	-0.18298	-0.19878	0.57987
Minimum	-0.14646	-0.12487	-0.12761	-0.12082	-0.13128	0.00000
Median	0.00138	0.00094	0.00094	0.00088	0.00085	0.00000
Maximum	0.10701	0.09109	0.08977	0.09066	0.09036	0.00048
Mean (annual)	8.79%	7.34%	7.36%	6.74%	6.76%	0.72%
StDev (annual)	33.02%	25.53%	25.57%	25.83%	26.12%	0.09%
CAGR	9.19%	7.62%	7.64%	6.97%	6.99%	0.72%
HPR	30.17%	24.63%	24.72%	22.42%	22.46%	2.18%

Tech ETF had higher return than S&P 500 ETF but at greater risk.

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HPR	30.17%	24.63%	24.72%	22.42%	22.46%	2.18%

ETFs had lower returns than benchmarks.

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Analysis

- Compute correlations.

Correlation matrix						
	IYW	VOO	SPX	VII	CRSPTMT	EEFR
IYW	1	0.925	0.927	0.926	0.927	-0.055
VOO	0.925	1	0.999	0.996	0.996	-0.045
SPX	0.927	0.999	1	0.996	0.997	-0.046
VII	0.926	0.996	0.996	1	0.999	-0.048
CRSPTMT	0.927	0.996	0.997	0.999	1	-0.049
EEFR	-0.055	-0.045	-0.046	-0.048	-0.049	1

Cash equivalents return uncorrelated with stock market.

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Analysis

- Compute correlations.

Correlation matrix						
	IYW	VOO	SPX	VII	CRSPTMT	EEFR
IYW	1	0.925	0.927	0.926	0.927	-0.055
VOO	0.925	1	0.999	0.996	0.996	-0.045
SPX	0.927	0.999	1	0.996	0.997	-0.046
VII	0.926	0.996	0.996	1	0.999	-0.048
CRSPTMT	0.927	0.996	0.997	0.999	1	-0.049
EEFR	-0.055	-0.045	-0.046	-0.048	-0.049	1

Stock market indexes are highly correlated.

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Analysis

- Compute correlations.

Correlation matrix						
	IYW	VOO	SPX	VII	CRSPTMT	EEFR
IYW	1	0.925	0.927	0.926	0.927	-0.055
VOO	0.925	1	0.999	0.996	0.996	-0.045
SPX	0.927	0.999	1	0.996	0.997	-0.046
VII	0.926	0.996	0.996	1	0.999	-0.048
CRSPTMT	0.927	0.996	0.997	0.999	1	-0.049
EEFR	-0.055	-0.045	-0.046	-0.048	-0.049	1

Tech ETF is less than perfectly correlated because it is not as well diversified.

- Tactical allocation or market timing strategy.
 - Your decision; not active manager's.
 - IYW is passive portfolio of tech stocks.

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Analysis

- Compute IYW performance measures.

Risk-adjusted performance				
Description	VOO	SPX	VII	CRSPTMT
<i>n</i>	756	756	756	756
Sharpe ratio - portfolio	0.0154	0.0154	0.0154	0.0154
Sharpe ratio - market	0.0163	0.0164	0.0147	0.0146
M-squared	0.0000	0.0000	0.0000	0.0000
Treynor ratio - portfolio	0.0003	0.0003	0.0003	0.0003
Treynor ratio - market	0.0003	0.0003	0.0002	0.0002
Jensen's alpha	0.0000	0.0000	0.0000	0.0000
Appraisal ratio	0.0007	0.0006	0.0047	0.0051
Sortino ratio - portfolio	0.0211	0.0211	0.0211	0.0211
Sortino ratio - market	0.0221	0.0222	0.0198	0.0196
M-squared - CAGR	-0.49%	-0.51%	0.37%	0.44%
Jensen's alpha - CAGR	0.15%	0.12%	0.94%	1.00%

When benchmarked to tradable S&P 500 (VOO ETF) or S&P 500 benchmark, poor risk-adjusted performance.

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Analysis

- Compute IYW performance measures.

Risk-adjusted performance				
Description	VOO	SPX	VTI	CRSPTMT
<i>n</i>	756	756	756	756
Sharpe ratio - portfolio	0.0154	0.0154	0.0154	0.0154
Sharpe ratio - market	0.0163	0.0164	0.0147	0.0146
M-squared	0.0000	0.0000	0.0000	0.0000
Treynor ratio - portfolio	0.0003	0.0003	0.0003	0.0003
Treynor ratio - market	0.0003	0.0003	0.0002	0.0002
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Appraisal ratio	0.0007	0.0006	0.0047	0.0051
Sortino ratio - portfolio	0.0211	0.0211	0.0211	0.0211
Sortino ratio - market	0.0221	0.0222	0.0198	0.0196
M-squared - CAGR	-0.49%	-0.51%	0.37%	0.44%
Jensen's alpha - CAGR	0.15%	0.12%	0.94%	1.00%

When benchmarked to tradable total market index (VTI ETF) or CRSPTMT benchmark, good risk-adjusted performance.

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Analysis

- Compute IYW performance measures.

Risk-adjusted performance				
Description	VOO	SPX	VTI	CRSPTMT
<i>n</i>	756	756	756	756
Sharpe ratio - portfolio	0.0154	0.0154	0.0154	0.0154
Sharpe ratio - market	0.0163	0.0164	0.0147	0.0146
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Treynor ratio - market	0.0003	0.0003	0.0002	0.0002
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Sortino ratio - portfolio	0.0211	0.0211	0.0211	0.0211
Sortino ratio - market	0.0221	0.0222	0.0198	0.0196
M-squared - CAGR	-0.49%	-0.51%	0.37%	0.44%
Jensen's alpha - CAGR	0.15%	0.12%	0.94%	1.00%

Ratios like Sharpe ratio and Treynor ratio have little economic appeal.

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Analysis

- Compute IYW performance measures.

Risk-adjusted performance				
Description	VOO	SPX	VTI	CRSPTMT
<i>n</i>	756	756	756	756
Sharpe ratio - portfolio	0.0154	0.0154	0.0154	0.0154
Sharpe ratio - market	0.0163	0.0164	0.0147	0.0146
M-squared	0.0000	0.0000	0.0000	0.0000
Treynor ratio - portfolio	0.0003	0.0003	0.0003	0.0003
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Sortino ratio - portfolio	0.0211	0.0211	0.0211	0.0211
Sortino ratio - market	0.0221	0.0222	0.0198	0.0196
M-squared - CAGR	-0.49%	-0.51%	0.37%	0.44%
Jensen's alpha - CAGR	0.15%	0.12%	0.94%	1.00%

Investors/managers want outperformance expressed in annualized return percentage.

$$CAGR = e^{daily\ outperformance \times 252} - 1$$

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Attributes of benchmark

- Must be all-inclusive.
 - Should not use S&P 500.
 - Leaves VTI and CRSPTMT.
- Must be investable (or tradable).
 - Leaves VTI.
- Must account for costs.
 - VTI returns are after management fees (i.e., expense ratio).
- Must have liquid market.
 - Bid/ask spread is \$0.01 (or less).

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Isolating alpha

- Suppose you want gains from tech stocks, not gains or losses from overall market.
 - Can you isolate returns from your tactical decision?

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Isolating alpha

- Measure relative market risk, β , using VTI.
- Support sheet: Portable alpha
 - Beta is 1.18355.

Excess return regression - VTI	
Benchmark index	IYW
n	756
α	0.00004
$s(\alpha)$	0.00029
β	1.18355
$s(\beta)$	0.01759
R-squared	0.8573
Adj. R-squared	0.8571
Std error of estimate	0.00786
t -ratio ($H_0: \alpha=0$)	0.13
t -ratio ($H_0: \beta=1$)	10.44

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Isolating alpha

- Create hedged portfolio by short selling VTI.
 - Call it alpha portfolio.
 - Amount to short sell is 1.18355 VTI return.
 - Invest cash generated in cash equivalents.
 - Support sheet: Portable alpha

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Isolating alpha

- Consider short selling market ETF against your tactical allocation to IYW.
 - Excess return regress produces estimates of alpha and beta.
- By OLS regression assumptions,

$$R_{S,t} - R_{F,t} = \alpha_S + \beta_S (R_{M,t} - R_{F,t}) + \varepsilon_{S,t}$$

$$\bar{R}_S = \hat{\alpha} + \bar{R}_F + \hat{\beta}_S (\bar{R}_M - \bar{R}_F)$$

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Isolating alpha

- Hedged portfolio return is

$$\begin{aligned} \bar{R}_S &= \hat{\alpha} + \bar{R}_F + \hat{\beta}_S (\bar{R}_M - \bar{R}_F) - \bar{R}_F - \hat{\beta}_S (\bar{R}_M - \bar{R}_F) \\ &= \hat{\alpha} \end{aligned}$$

- Net amount invested is 0.
 - Purchase of IYW is fully financed with short sale of VTI.
 - Have funds to invest.

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Isolating alpha

- Regress alpha portfolio return on VTI.

Excess return regression - VTI

Benchmark index	Alpha
<i>n</i>	756
<i>α</i>	0.00004
<i>s(α)</i>	0.00029
<i>β</i>	0.00000
<i>s(β)</i>	0.01759
R-squared	0.0000
Adj. R-squared	-0.0013
Std error of estimate	0.00786

No stock market risk.

<i>t</i> -ratio ($H_0: \alpha=0$)	0.13
<i>t</i> -ratio ($H_0: \beta=0$)	0.00

Isolated and retained alpha from tactical decision.

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Portable alpha

- For active funds, alpha measures manager's skill.
 - Skill can be isolated by creating hedged portfolio:
 - Short sell passively managed market index ETF.
 - Effectively turns portfolio into investment whose expected excess return equals alpha with no investment.
 - Sell futures on market index (usually less expensive).
 - Effectively turns portfolio into investment whose expected excess return equals alpha plus risk-free rate earned on original investment amount.

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Portable alpha benchmarks

- Portable alpha requires actively-traded market price risk factor.
 - For stocks, use:
 - All-inclusive, inexpensive, total market index ETP like VTI.
 - Futures contract written on broad-based index like S&P 500 futures (CME).
 - For bonds, use:
 - All-inclusive, inexpensive, total market index ETP like BND.
 - Futures contract written on broad-based index like Treasury bond futures (CBT).

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BOND

- Illustration: Currently hold Pimco’s BOND ETF.
 - Believe intermediate- and long-term interest rates are about to fall.
 - Want to immunize bond price risk exposure while maintaining Pimco’s ability to generate positive alpha in BOND.
 - Support file: BOND.xlsx

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BOND

- Illustration: Check BOND’s past returns.

Summary statistics				
Description	BOND	BND	VTI	EFFR
<i>n</i>	2,701	2,701	2,701	2,701
Mean (daily)	0.000106	0.000062	0.000469	0.000034
StDev (daily)	0.002887	0.003080	0.011199	0.000056
Skewness	-1.8232	-1.5591	-0.8256	3.3457
Autocorrelation	0.0474	0.0067	-0.1060	0.5120
Minimum	-0.0428	-0.0559	-0.1208	0.0000
Median	0.0003	0.0002	0.0007	0.0000
Maximum	0.0198	0.0413	0.0907	0.0005
Mean (annual)	2.68%	1.57%	11.81%	0.86%
StDev (annual)	4.58%	4.89%	17.78%	0.09%
CAGR	2.71%	1.59%	12.53%	0.87%
HPR	33.24%	18.36%	254.49%	9.70%

BOND has substantially greater return and lower risk than BND.

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BOND

□ Illustration: Check BOND's correlations.

Correlation matrix				
	BOND	BND	VII	EFFR
BOND	1	0.863	0.104	-0.016
BND	0.863	1	0.073	-0.024
VII	0.104	0.073	1	-0.036
EFFR	-0.016	-0.024	-0.036	1

Only way to earn abnormal return is to take on residual risk.

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BOND

□ Illustration: Compute risk-adjusted performance.

Risk-adjusted performance	
Description	BOND
<i>n</i>	2,701
Sharpe ratio - portfolio	0.024916
Sharpe ratio - market	0.009133
M-squared	0.000046
Treynor ratio - portfolio	0.000089
Treynor ratio - market	0.000028
Jensen's alpha	0.000049
Appraisal ratio	0.033747
Sortino ratio - portfolio	0.033183
Sortino ratio - market	0.012352
M-squared - CAGR	1.16%
Jensen's alpha - CAGR	1.25%

BOND substantially outperforms on annualized basis.

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BOND

- Illustration: Short sell BND against long position in BOND to generate daily alpha return.

Regressions on BND		
	BOND	Alpha
<i>n</i>	2,701	2,701
α	0.000049	0.000049
$s(\alpha)$	0.000028	0.000028
β	0.809242	0.000000
$s(\beta)$	0.009103	0.009103
R-squared	0.7454	0.0000
Adj. R-squared	0.7453	-0.0004
Std error of estimate	0.00146	0.00146

Run regression of excess return of BOND on excess return of BND.

For every dollar invested in BOND, sell 0.809242 dollars in BND.

$$\alpha_t = XR_{BOND,t} - 0.809242 \times XR_{BND,t}$$

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BOND

- Illustration: Short sell BND against long position in BOND to generate daily alpha return.

Regressions on BND		
	BOND	Alpha
<i>n</i>	2,701	2,701
α	0.000049	0.000049
$s(\alpha)$	0.000028	0.000028
β	0.809242	0.000000
$s(\beta)$	0.009103	0.009103
R-squared	0.7454	0.0000
Adj. R-squared	0.7453	-0.0004
Std error of estimate	0.00146	0.00146

No long-term interest rate risk (i.e., bond market risk) remains.

Outperformance does.

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Portable alpha

- Alpha portfolio has no investment and no market risk but has abnormal return.
 - Alpha portfolio is called alpha engine in practice.
 - Term portable arises from fact that alpha engine can now be attached to another asset category.
 - Buy VTI with funds and will have abnormal stock market performance.
 - Why? Abnormal performance in high market cap stocks is unusual, and investment manager is rewarded on outperformance.

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BOND (continued)

- Buy VTI with investment funds (cash generated from short sale) while holding alpha portfolio.

Regression on VTI	
	Alpha+VTI
n	2,701
α	0.000045
$s(\alpha)$	0.000028
β	1.010690
$s(\beta)$	0.002496
R-squared	0.983801
Adj. R-squared	0.983795
Std error of estimate	0.001453

Why isn't alpha equal to 0.000049 and beta equal to one? (To be continued.)

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Lesson summary

- Active fund return is attributable to:
 - Market risk exposures: Betas (not just stock market)
 - Investment skills: Portable alpha
- Do not need manager to create market risk exposures.
 - Can create cheaply using futures or passive ETPs.

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Lesson summary

- Described four traditional risk-adjusted performance measures and their origins.
 - Two are based on total volatility.
 - Sharpe ratio
 - M-squared
 - Two are based on beta.
 - Treynor ratio
 - Jensen alpha

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Lesson summary

- Discussed two other measures are ad hoc, albeit intuitive.
 - One is based on excess return regression.
 - Treynor/Black appraisal ratio
 - One is based on ad hoc measure of risk.
 - Sortino ratio
- Introduced concept called portable alpha.
 - Direct by-product of Jensen's alpha.
 - Can transport alpha (i.e., outperformance) across asset categories.

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Lesson summary

- Understand source of portable alpha.
 - Why are prices incorrect and what forces them to correct?

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