EXPECTED RETURN AND RISK OF FUTURES CONTRACTS

Like other risky financial instruments, futures contracts have expected returns and risks that can be modeled within the capital asset pricing model (CAPM). The key to understanding exactly how lies in the relation between the rate of price change of a futures and the rate of return of its underlying asset.

Futures/asset return relation

To begin, we recall the net cost of carry relation,

$$F_t = S_t e^{(r-i)(T-t)} \tag{5.1}$$

where the subscript t has been added to denote a particular point in time prior to the contract's expiration. Taking the natural logarithm of both sides of (5.1) provides

$$\ln F_t = (r-i)(T-t) + \ln S_t \tag{5.2}$$

Now consider the transformed net cost of carry relation (5.2) an instant earlier in time at $t+1$ that is,

$$\ln F_{t-1} = (r-i)(T-t+1) + \ln S_{t-1} \tag{5.3}$$

Subtracting (5.3) from (5.2), we find that the continuous rate price change of the futures is

$$R_F \equiv RA_F \equiv \ln(F_t/F_{t-1}) = -(r-i) + \ln(S_t/S_{t-1}) \tag{5.4}$$

In equation (5.4), R_F is the rate of return on a futures contract and RA_F is its rate of price appreciation. We make this distinction to underscore the fact that the only income arising from holding a futures contract is price change. The rate of return from investment in the underlying asset, R_S, on the other hand, is the sum of two components—the continuous rate of price appreciation $RA_S \equiv \ln\left(S_t / S_{t-1}\right)$ and the income rate i. The relation between the random returns of the futures and its underlying asset is, therefore,

$$\tilde{R}_F = \tilde{R}_S - r \tag{5.5}$$

where tildes have been added to distinguish between what is uncertain (i.e., the returns on the futures and its underlying asset) from what is certain (i.e., the risk-free rate of interest).

Expected return/risk relation

With the return relation (5.5) in hand, the role of futures contracts within the CAPM is easily identified. First, note the expected return on a futures contract equals the expected return on the underlying asset less the risk-free rate of interest, that is,

$$E_F = E_S - r \tag{5.6}$$

Next, note that total risk (as measured by return variance or its square root, standard deviation) and market risk (as measured by beta) of the futures contract equal the total risk and the market risk of the underlying asset. The variance of futures return equals the variance of the asset return,

$$\mathrm{Var}(\tilde{R}_F) = \mathrm{Var}(\tilde{R}_S - r) = \mathrm{Var}(\tilde{R}_S) \tag{5.7}$$

and the beta of the futures contract equals the beta of the underlying asset,

$$\beta_F \equiv \frac{\mathrm{Cov}(\tilde{R}_F, \tilde{R}_M)}{\mathrm{Var}(\tilde{R}_M)} = \frac{\mathrm{Cov}(\tilde{R}_S - r, \tilde{R}_M)}{\mathrm{Var}(\tilde{R}_M)} = \frac{\mathrm{Cov}(\tilde{R}_S, \tilde{R}_M)}{\mathrm{Var}(\tilde{R}_M)} \equiv \beta_S \tag{5.8}$$

Hence, while the risks of the futures contract are the same as those of the underlying asset, the expected return of the futures is below the expected return of the underlying asset by an amount equal to the risk-free rate of interest.

Now let us recall the security market line from the CAPM. The expected return of an asset equals the risk-free return plus a market risk premium, that is,

$$E_S = r + (E_M - r)\beta_S \tag{5.9}$$

Substituting (5.6) and (5.8) into (5.9), we find that the expected return on the futures is

$$E_F = (E_M - r)\beta_F = (E_M - r)\beta_S \tag{5.10}$$

While on first appearance the relation (5.10) may seem perplexing, it makes a good deal sense intuitively. In buying the asset, we are buying two things—the risk-free asset and a risk premium. We are entitled to the rate of return on the risk-free asset, *r*, because we have funds tied up in the asset, independent of its risk level. In addition, we are entitled to the risk premium associated with holding the asset, $\left(E_M - r\right)\beta_S$, because we have put our investment at risk. In buying the futures, we have accepted only the risk and, therefore, are entitled to receive only the risk premium. With no funds tied up, we have no right to any risk-free return.

Relation to Net Cost of Carry

The net cost of carry relation implies that being long the asset and short a futures is equivalent to being long risk-free bonds. Equations (5.9) and (5.10) confirm this result. Being long the asset means that we expect rate of return E_S and being short the futures means that we expect rate of return E_F. Thus the net portfolio return from being long the asset and short the futures equals the risk-free rate of interest,

$$E_S - E_F = r + (E_M - r)\beta_S - (E_M - r)\beta_S = r \tag{5.11}$$

The risk premium associated with buying the asset is exactly offset by the risk premium associated with selling the futures.

Based on (5.11), various pairings of instruments provide synthetic replications. Suppose, for example, we buy risk-free bonds and buy a futures. The expected portfolio return is exactly equal to that of the underlying asset, that is,

$$r + E_F = r + (E_M - r)\beta_S = E_S \tag{5.12}$$

Table 1 summarizes all possible pairings. The intuition is simple. Buying or selling a futures is the same as buying or selling a risk premium. Buying and selling an asset, on the other hand, means buying and selling a portfolio that consists of the risk-free asset and a risk premium. Note that, if the risk premium of the asset happens to equal zero, the expected rate of price change in the futures is zero.

Table 1: Perfect substitutes implied by the capital asset pricing model

Position 1		Position 2
Buy asset/sell forward	=	Buy risk-free bonds (lend)
Buy risk-free bonds (lend)/buy forward	=	Buy asset
Buy asset/sell risk-free bonds (borrow)	=	Buy forward
Sell asset/buy forward	=	Sell risk-free bonds (borrow)
Sell risk-free bonds (borrow)/sell forward	=	Sell asset
Sell asset/buy risk-free bonds (lend)	=	Sell forward

Futures as Predictor of Expected Asset Price

The relation between expected return and risk of the asset and the futures also provides us with insight regarding the relation between the current futures price and the expected asset price when the futures expires at time T. To see this, consider committing to buy the asset at time T. The present value of the expected asset price is

$$S = E(\tilde{S}_T)e^{-E_S T}$$

where E_S is the asset's expected risk-adjusted rate of return. On the other hand, consider committing to buy the asset at time T by buying a futures contract today at price F. Since F is paid at time T and is certain, the present value of this obligation is Fe^{-rT}. Since both quantities represent the same thing—the present value of one unit of the commodity at time T—they should be equal in value. Thus, the current futures price may be written

$$F = E(\tilde{S}_T)e^{-(E_S - r)T} \tag{5.13}$$

The structure of (5.13) says that the difference between the futures price and the expected asset price is nonzero. This means that the futures price is a biased predictor of the expected asset price. If the risk premium is positive, as is usually the case, the futures price is a downward biased predictor. The only instance in which the futures price is an unbiased predictor of the expected future asset price is where the risk premium of the asset equals 0.