ESTIMATING MEAN REVERSION

Many price series (e.g., commodities, and volatility) follow mean-reverting processes (i.e., have mean reversion). For commodities, for example, if the price is above the long-run mean, supply increases, and demand declines, bringing the price down. Conversely, if the price is below the long-run mean, supply decreases, and demand increases, causing the price to rise.

The simple standard discrete-time model of a mean reverting process is

$$S_{t+1} = S_t + \kappa\left(\mu - S_t\right) + \varepsilon_{t+1} \tag{1}$$

where S_t is the current value of the process, μ is the long run mean, κ is the speed of adjustment coefficient, and ε_{t+1} is a random shock that is (a) independent of S_t and of previous ε_t, (b) normally distributed, (c) mean 0, and (d) constant variance. To estimate the model (1), transform it to:

$$\Delta S_{t+1} = \kappa\mu - \kappa S_t + \varepsilon_{t+1} \ . \tag{2}$$

Then, use OLS regression to estimate (2),

$$\Delta S_{t+1} = \beta_0 + \beta_1 S_t + \varepsilon_{t+1} \ . \tag{3}$$

The regression produces estimates of the intercept and slope ($\hat{\beta}_0$ and $\hat{\beta}_1$) and their standard errors ($\hat{\sigma}_{\beta_0}$ and $\hat{\sigma}_{\beta_1}$). Comparing (2) and (3), we see the estimated slope provides the estimate of the speed of adjustment coefficient, $\hat{\kappa} = -\hat{\beta}_1$ and the intercept, in combination with the speed of adjustment coefficient, provides an estimate of the long-run mean, $\hat{\mu} = \dfrac{\hat{\beta}_0}{\kappa}$ The results may be interpreted as follows:

1) If $\hat{\beta}_1 > 0$, then no mean reversion is apparent.
2) If $\hat{\beta}_1 < 0$, then process is mean-reverting,
    a. If $\hat{\beta}_1$ is significant, then mean reversion is present.
3) If $\hat{\beta}_1$ is significantly negative and $\hat{\beta}_0$ is significantly different from 0, then
    a. the long run mean is $\hat{\mu} = \hat{\beta}_0 / \hat{\kappa}$
    b. the estimated standard deviation of the price change series is the standard error of the estimate from the regression $\hat{\sigma} = \hat{\sigma}_\varepsilon$.