

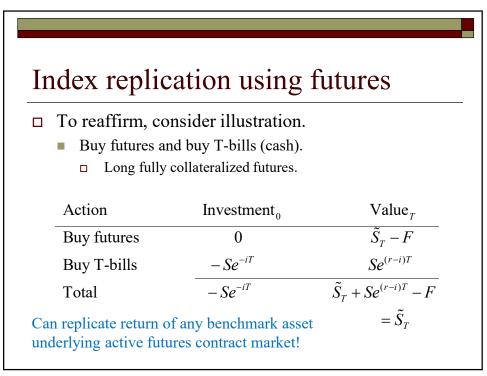
Index replication using futures □ Key assumption: Arbitragers will remove all freemoney opportunities that appear. No human intervention; entirely mechanical. Program trading What markets? Stock markets Foreign currency markets Bond markets Derivatives markets

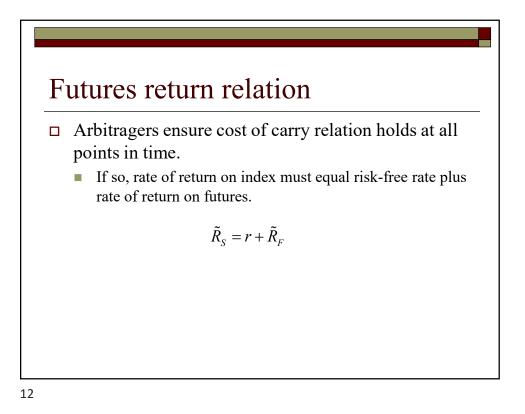
□ Cost of carry relation implies:

Position 1		Position 2
Buy asset/sell futures	=	Buy risk-free bonds (lend)
Buy risk-free bonds (lend)/buy futures	=	Buy asset
Buy asset/sell risk-free bonds (borrow)	=	Buy futures
Sell asset/buy futures	=	Sell risk-free bonds (borrow)
Sell risk-free bonds (borrow)/sell futures	=	Sell asset
Sell asset/buy risk-free bonds (lend)	=	Sell futures

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Index replication using futuresCost of carry relation implies:			
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Buy risk-free bonds (lend)/buy futures	=	Buy asset	
Buy asset/sell risk-free bonds (borrow)	=	Buy futures	
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Sell risk-free bonds (borrow)/sell futures	=	Sell asset	
Sell asset/buy risk-free bonds (lend)	=	Sell futures	







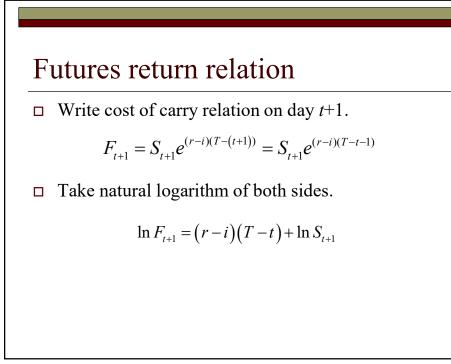
 \Box Write cost of carry relation on day *t*.

 $F_t = S_t e^{(r-i)(T-t)}$

□ Take natural logarithm of both sides.

$$\ln F_t = (r-i)(T-t) + \ln S_t$$

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Futures return relation

Difference ln prices to price appreciation and then return.

$$\ln(F_{t+1}) - \ln(F_{t}) = \ln(F_{t+1} / F_{t}) = -(r - i) + \ln(S_{t+1} / S_{t})$$
$$RA_{F} \equiv R_{F} = -(r - i) + RA_{S}$$
$$R_{F} = (RA_{S} + i) - r$$
$$R_{F} = R_{S} - r$$

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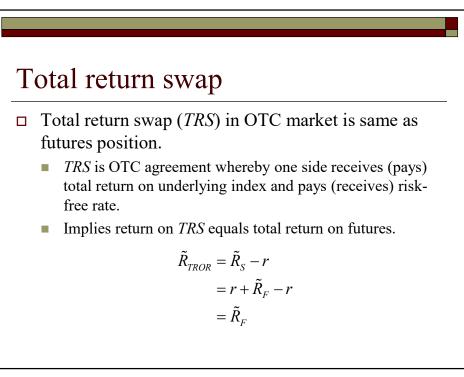
Futures return relation

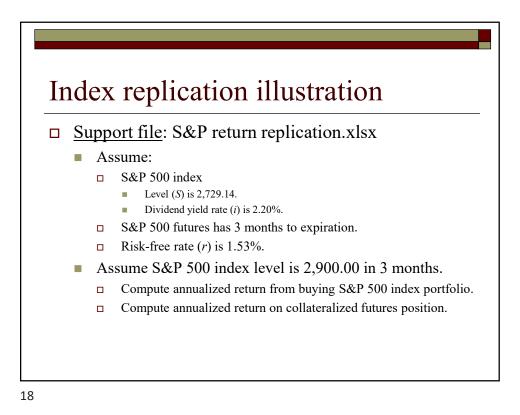
□ If cost of carry relation holds,

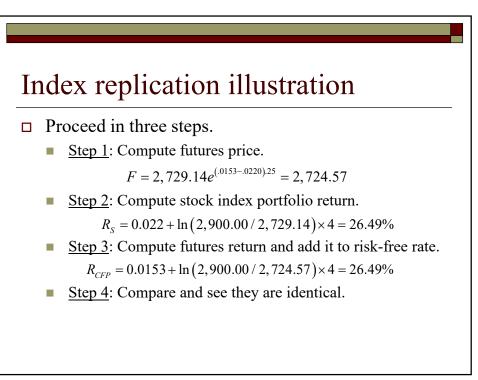
$$F_t = S_t e^{(r-i)(T-t)}$$

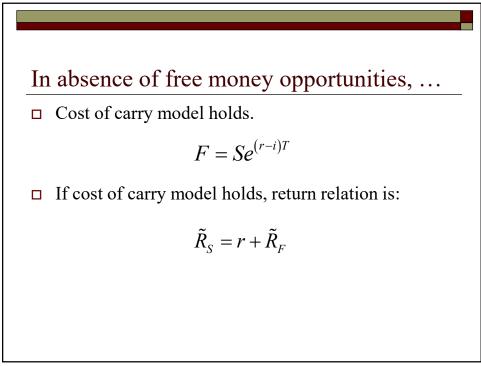
benchmark index return equals return on fully collateralized futures position (i.e., risk-free rate plus index futures return).

$$\tilde{R}_{S} = r + \tilde{R}_{F}$$











- □ Second-generation ETPs are based on fullycollateralized futures positions.
 - Assuming active arbitrage between futures and cash markets, issuer can buy T-bills and equal notional amount of futures to earn benchmark index return.
 - Usually used in markets where investing in security is expensive or restricted.