AIM 03

Fully-collateralized futures products

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Applied investment management

- Context:
  - First-generation or traditional ETPs held actual (physical) securities that mimic benchmark.
    - Full replication – Buy all securities in benchmark index.
    - Sampling and optimization – Buy subset that mimics benchmark index almost exactly.
  - Second-generation products hold futures and cash equivalents to obtain 1x return on underlying benchmark.
    - Not typically used for financial securities like stocks and bonds or physical assets like gold.
    - Used to create new asset classes like commodities and volatility.

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Fully-collateralized futures products

- Purpose:
  - Explain structure of fully collateralized futures ETP.
  - Prove fully collateralized futures position has identical return to holding benchmark index.
    - Index return = Risk-free return + Index futures return

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Index replication using futures

- Key concept is cost of carry relation is

$$F = Se^{(r-i)T}$$

  - F is futures price
  - S is price of index underlying futures
  - r is interest rate
  - i is income rate
    - r and i are assumed to be constant, continuous rates.
  - T is time to expiration date of futures

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Costless arbitrage portfolio

- Create costless arbitrage portfolio using security, futures, and cash (i.e., risk-free borrowing or lending):
  - Cost is zero (costless).
  - Risk is zero (arbitrage).

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Index replication using futures

- Cost of carry relation is:

$$F = Se^{(r-i)T}$$

  - Is called no-arbitrage price relation.
    - Holds because no "free-money" opportunities in efficiently functioning marketplace.
  - Term arbitrage is often misapplied in practice.
    - E.g., risk arbitrage

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Costless arbitrage portfolio

Action	Investment_0	Value_T
Buy futures	0	$\tilde{S}_T - F$
Sell index portfolio	Se^{-iT}	$-\tilde{S}_T$
Buy T-bills	$-Se^{-iT}$	$Se^{(r-i)T}$
Total	0	$Se^{(r-i)T} - F$

No investment and outcome is known with certainty.

$$Se^{(r-i)T} - F = 0 \text{ or } F = Se^{(r-i)T}$$

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Index replication using futures

- Key assumption: Arbitragers will remove all free-money opportunities that appear.
  - No human intervention; entirely mechanical.
    - Program trading
  - What markets?
    - Stock markets
    - Foreign currency markets
    - Bond markets
    - Derivatives markets

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Index replication using futures

- Cost of carry relation implies:

Position 1		*Position 2*
Buy asset/sell futures	=	Buy risk-free bonds (lend)
Buy risk-free bonds (lend)/buy futures	=	Buy asset
Buy asset/sell risk-free bonds (borrow)	=	Buy futures
Sell asset/buy futures	=	Sell risk-free bonds (borrow)
Sell risk-free bonds (borrow)/sell futures	=	Sell asset
Sell asset/buy risk-free bonds (lend)	=	Sell futures

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Index replication using futures

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Buy risk-free bonds (lend)/buy futures	=	Buy asset
Buy asset/sell risk-free bonds (borrow)	=	Buy futures
Sell asset/buy futures	=	Sell risk-free bonds (borrow)
Sell risk-free bonds (borrow)/sell futures	=	Sell asset
Sell asset/buy risk-free bonds (lend)	=	Sell futures

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Index replication using futures

- To reaffirm, consider illustration.
  - Buy futures and buy T-bills (cash).
    - Long fully collateralized futures.

Action	Investment_0	Value_T
Buy futures	0	$\tilde{S}_T - F$
Buy T-bills	$-Se^{-iT}$	$Se^{(r-i)T}$
Total	$-Se^{-iT}$	$\tilde{S}_T + Se^{(r-i)T} - F$
		$= \tilde{S}_T$

Can replicate return of any benchmark asset underlying active futures contract market!

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Futures return relation

- Arbitragers ensure cost of carry relation holds at all points in time.
  - If so, rate of return on index must equal risk-free rate plus rate of return on futures.

$$\tilde{R}_S = r + \tilde{R}_F$$

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Futures return relation

- Write cost of carry relation on day t.

$$F_t = S_t e^{(r-i)(T-t)}$$

- Take natural logarithm of both sides.

$$\ln F_t = (r-i)(T-t) + \ln S_t$$

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Futures return relation

- Write cost of carry relation on day t+1.

$$F_{t+1} = S_{t+1} e^{(r-i)(T-(t+1))} = S_{t+1} e^{(r-i)(T-t-1)}$$

- Take natural logarithm of both sides.

$$\ln F_{t+1} = (r-i)(T-t) + \ln S_{t+1}$$

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Futures return relation

- Difference ln prices to price appreciation and then return.

$$\ln(F_{t+1}) - \ln(F_t) = \ln(F_{t+1}/F_t) = -(r-i) + \ln(S_{t+1}/S_t)$$

$$RA_F \equiv R_F = -(r-i) + RA_S$$

$$R_F = (RA_S + i) - r$$

$$R_F = R_S - r$$

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Futures return relation

- If cost of carry relation holds,

$$F_t = S_t e^{(r-i)(T-t)}$$

benchmark index return equals return on fully collateralized futures position (i.e., risk-free rate plus index futures return).

$$\tilde{R}_S = r + \tilde{R}_F$$

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Total return swap

- Total return swap (*TRS*) in OTC market is same as futures position.
  - *TRS* is OTC agreement whereby one side receives (pays) total return on underlying index and pays (receives) risk-free rate.
  - Implies return on *TRS* equals total return on futures.

$$\begin{aligned}\tilde{R}_{TROR} &= \tilde{R}_S - r \\ &= r + \tilde{R}_F - r \\ &= \tilde{R}_F\end{aligned}$$

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Index replication illustration

- Support file: S&P return replication.xlsx
  - Assume:
    - S&P 500 index
      - Level (S) is 2,729.14.
      - Dividend yield rate (i) is 2.20%.
    - S&P 500 futures has 3 months to expiration.
    - Risk-free rate (r) is 1.53%.
  - Assume S&P 500 index level is 2,900.00 in 3 months.
    - Compute annualized return from buying S&P 500 index portfolio.
    - Compute annualized return on collateralized futures position.

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Index replication illustration

- Proceed in three steps.
  - Step 1: Compute futures price.

$$F = 2,729.14e^{(.0153-.0220).25} = 2,724.57$$

  - Step 2: Compute stock index portfolio return.

$$R_S = 0.022 + \ln(2,900.00 / 2,729.14) \times 4 = 26.49\%$$

  - Step 3: Compute futures return and add it to risk-free rate.

$$R_{CFP} = 0.0153 + \ln(2,900.00 / 2,724.57) \times 4 = 26.49\%$$

  - Step 4: Compare and see they are identical.

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In absence of free money opportunities, …

- Cost of carry model holds.

$$F = Se^{(r-i)T}$$

- If cost of carry model holds, return relation is:

$$\tilde{R}_S = r + \tilde{R}_F$$

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Lesson summary

- Second-generation ETPs are based on fully-collateralized futures positions.
  - Assuming active arbitrage between futures and cash markets, issuer can buy T-bills and equal notional amount of futures to earn benchmark index return.
  - Usually used in markets where investing in security is expensive or restricted.

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